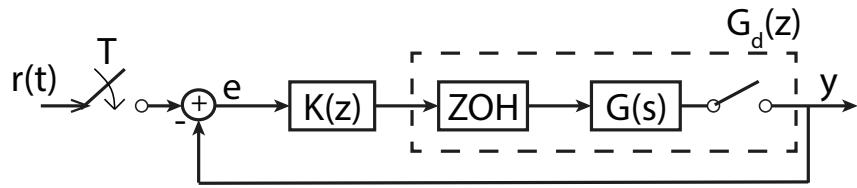


# 16.06 Principles of Automatic Control

## Recitation 13



$$\frac{E}{R} = \frac{1}{1 + KG} \rightarrow E = \frac{E}{R} \cdot R$$

$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s)$ , in continuous time.

$$e_{ss} = \lim_{k \rightarrow \infty} e[k] = \lim_{z \rightarrow 1} (z - 1) E(z)$$

For a step input,  $r = \sigma(t)$ .

C	D
$e_{ss} = \frac{1}{1 + K(0)G(0)}$	$e_{ss} = \frac{1}{1 + K_d(1)G_d(1)}$
$K_p = K(0)G(0)$	$K_p = K_d(1)G_d(1)$

Given a type I system:

$$L(z) = G_d(z) K_d(z) = \frac{L_0(z)}{z - 1}$$

Find  $K_v$ :

$$\begin{aligned} \frac{E(z)}{R(z)} &= \frac{1}{1 + K_d(z)G_d(z)} \\ E(z) &= \frac{E(z)}{R(z)} \cdot R(z) = \frac{1}{1 + \frac{L_0(z)}{z-1}} \cdot \frac{T_z}{(z-1)^2} \end{aligned}$$

where  $T_z$  is unit ramp from the table.

Therefore,

$$\begin{aligned}
 e_{ss} &= \lim_{z \rightarrow 1} (z - 1) \frac{1}{1 + \frac{L_0(z)}{z-1}} \cdot \frac{T_z}{(z - 1)^2} \\
 &= \lim_{z \rightarrow 1} \frac{T_z}{z - 1 + L_o(z)} \\
 &= \frac{T}{L_0(1)} \\
 &= \frac{1}{K_v}
 \end{aligned}$$

$$\Rightarrow K_v = \frac{L_o(1)}{T}, \text{ in discrete time.}$$

Using the w-transform:

$$\text{Take } \frac{L_0(z)}{z - 1} = \frac{L_0\left(\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}\right)}{\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}} - 1} = \frac{L_0\left(\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}\right)\left(1 - \frac{wT}{2}\right)}{wT}$$

$$\begin{aligned}
 \therefore K_v &= \lim_{w \rightarrow 0} \frac{L_0\left(\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}\right)\left(1 - \frac{wT}{2}\right)}{wT} \cdot w \\
 &= \frac{L_0(1)}{T}
 \end{aligned}$$

The w-transform preserves  $K_p$  and  $K_v$ .

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