Chapter 5

Nonlinear control of satellites

5.1 Attitude control with thrusters

See Excerpts from A. E. Bryson: Control of Spacecraft and Aircraft, 1990 lecture notes (with the authorization of the author).

5.2 Nonlinear attitude control with spin

5.2.1 On the advantages and drawbacks of spin stabilization

Before getting into details, let us outline why spinning a body is a great stabilizing idea for axially symmetric aircraft: assume a body is not spinning and is subject to a constant torque disturbance. Then its angular velocity will grow linearly with time and its angular deviation will grow as t^2. Assume now the same body is spun at high speed, and the reference coordinates are fixed. Then a torque applied to the system will make the system precess at constant angular speed. Thus the angle deviation from original now grows only linearly with time. It should be noted that spin stabilization has been used on many types of compliant devices, including satellites, Saturn V launchers as well as gun bullets.

Among the drawbacks associated to spin stabilization is the *nutation* phenomenon: if the angular velocity vector is not exactly aligned with the main axis about which the satellite is rotated, then the satellite will wobble: This is the *nutation* phenomenon, which needs to be controlled via adequate means. Like attitude control, linear control laws may be (very effectively) used for nutation control. If thrusters are to be used, once again nutation may be most effectively damped using bang-bang, nonlinear control laws.

5.2.2   Equations of motion

If one does not care about final attitude yet, nutation control is best expressed in body coordinates. Assuming the spacecraft is symmetric with respect to its axis of rotation, we can choose a body-fixed coordinate set (x, y, z), with the satellite spinning about its z-axis. Let the angular speed vector ω be described by its three components p, q, r, and let the axial inertia be noted I_s and its tranverse inertia be I_t. Then, noting H the angular momentum of the satellite expressed in body coordinates, the equations for the dynamics of the satellite are

$$\frac{d}{dt} H \mid_{\text{b}} = Q - \omega \times H$$

where $\times$ is the usual vector product. Written componentwise, we obtain the three simultaneous equations

$$\frac{d}{dt} p = Q_x/I_t + (1 - I_s/I_t)qr$$

$$\frac{d}{dt} q = Q_y/I_t - (1 - I_s/I_t)pr$$

$$\frac{d}{dt} r = Q_z/I_s.$$

Thus, depending on whether the satellite has its axial inertia coëfficient is smaller than or greater than its transversal cëfficient, the resulting nutation will either be slower of faster than the actual angular speed of the satellite (the best way to convince yourself is to try it with simple objects).

5.2.3 Control of Nutation ($I_t > I_s$)

Assume that one differential thruster (one that can fire in two opposite directions) is available to produce moments about the x or y axis. In the simplest satellite systems, this thruster will be tied to the satellite and rotating with it. Thus, up to a fixed rotation, we may as well assume the produced torque is along the x-axis only, such that the resulting equations of motion are now

$$\frac{d}{dt}p = Q_x/I_t + (1 - I_s/I_t)qr$$

$$\frac{d}{dt}q = -(1 - I_s/I_t)pr$$

$$\frac{d}{dt}r = 0.$$

The last equation tells us that the spin speed is constant, such that the spin stabilization problem reduces to studying the second-order system with the generic form

$$\frac{d}{dt}p = \lambda q + u$$

$$\frac{d}{dt}q = -\lambda p, \quad \lambda > 0.$$

When no torque is applied, the motion described by p and q is a circle centered around 0. When positive control $u = u_0$ is applied, the motion is a circle centered around $(0, -u_0/\lambda)$, whereas when negative control is applied, the motion is a circle centered around $(0, u_0/\lambda)$. The control strategy here is to keep p and q within a given range $[-s; +s]$. One way of doing so is to apply $u = -u_0$ when $p > s$ and $u = u_0$ when $p < -s$; the resulting motion is shown in Fig. 5.1. Once again, undesired chattering phenomena may occur as shown in Fig. 5.2, such that the use of Schmitt triggers is recommended, to yield the phase portrait as shown in Fig. 5.3. The main drawback for these controllaws is that the system's performance is actually never better than $|p| \leq s$. A strategy that avoids this problem in principle is to make the switching lines go through 0, as shown in 5.4. Note however that switching delays and noise will always limit how near we can approach 0.
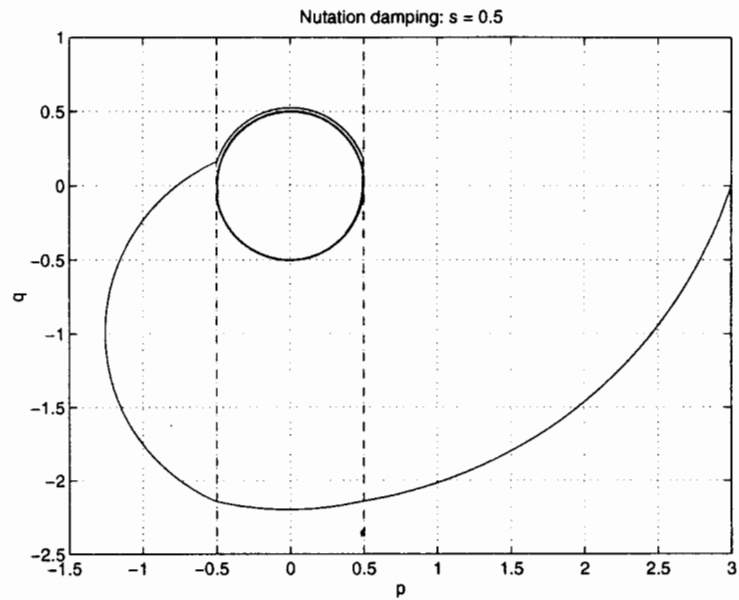
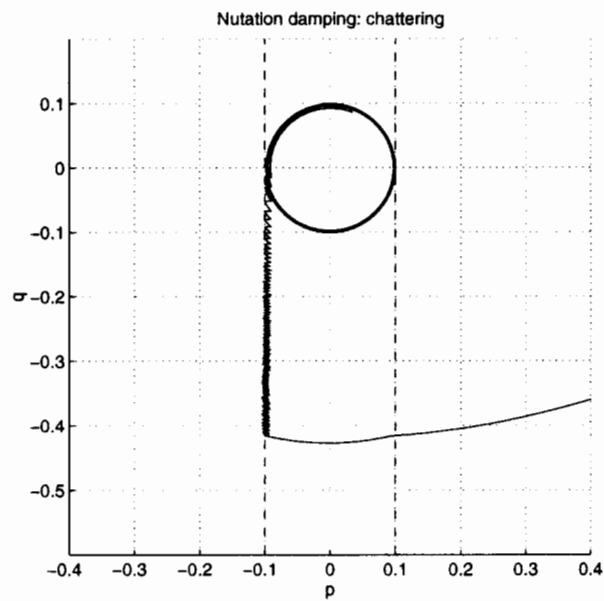Figure 5.1: Phase portrait for active nutation damping.

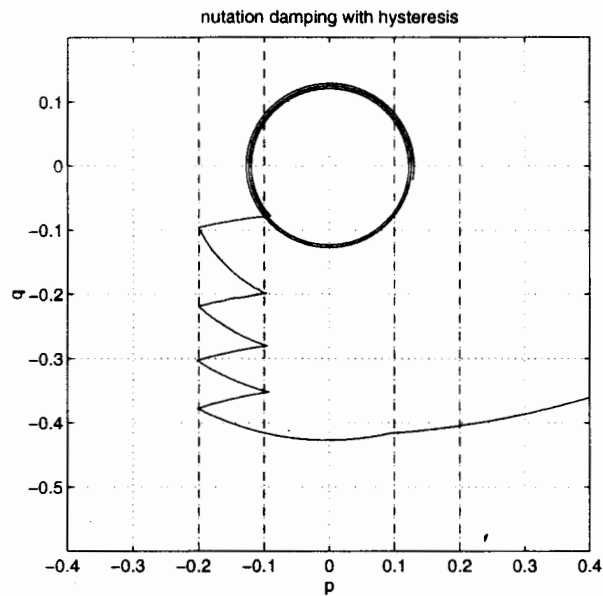Figure 5.2: Sliding phenomenon for active nutation damping.

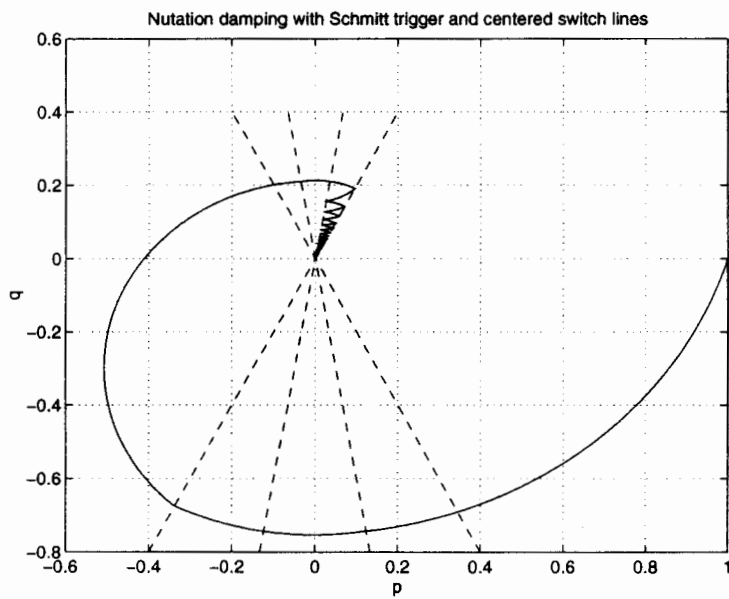Figure 5.3: Active nutation damping with Schmitt trigger.

Figure 5.4: Active nutation damping with Schmitt trigger and centered switch lines.

Among the advantages of nutation damping, we find the possibility to stabilize around axis of minimum inertia (why are these unstable?). Drawbacks of nutation damping (via thrusters) is that it changes the total angular momentum, thus the satellite orientation (might as well be seen as a random process). So this type of nutation damping needs to be completed by reorientation maneuvers.

5.3 Stabilization of translational motions of spacecraft

See A. E. Bryson: *Control of spacecraft and aircraft: 1990 course notes*