APPENDIX D

TABLE OF TWO-SINUSOID-INPUT DESCRIBING FUNCTIONS (TSIDFs)

The TSIDF can be represented by either of the following integral expressions (cf. Sec. 5.1)

$$N_B(A,B) = \frac{1}{2\pi^2 B}\int_{-\pi}^{\pi} d\psi_2 \sin\psi_2 \int_{-\pi}^{\pi} d\psi_1\, y(A\sin\psi_1 + B\sin\psi_2)$$

or

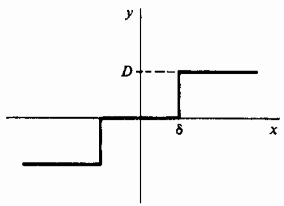
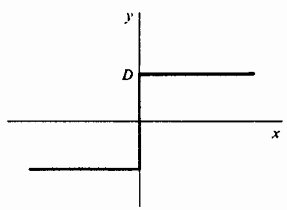
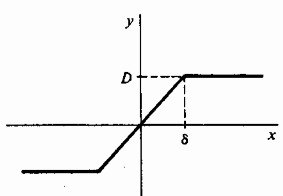
$$N_B(A,B) = \frac{j}{\pi B}\int_{-\infty}^{\infty} du\, J_0(Au)J_1(Bu)\int_{-\infty}^{\infty} dx\, y(x)e^{-jux}$$

where J_0 and J_1 are the Bessel functions of orders 0 and 1, respectively. Use of this table is facilitated by application of the relationship

$$N_A(A,B) = N_B(B,A)$$

All entries are for the case of nonharmonically related sinusoids, corresponding to the above integral formulations.

TABLE OF TWO-SINUSOID-INPUT DESCRIBING FUNCTIONS (TSIDFs) *(Continued)*

Nonlinearity	Comments	$N_B(A,B)$
 3. Relay with dead zone	${}_2F_1$ is the gaussian hypergeometric function. See Gibson and Sridhar, Ref. 10 of Chap. 5. See Fig. D.1	$\frac{4D}{\pi}\sum_{m=0}^{\infty}\frac{m\cos\left(2m\sin^{-1}\frac{\delta}{B}\right)}{\sqrt{1-\left(\frac{\delta}{B}\right)^2}}\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma(m+n)}{n!\,(m-n)!\,\Gamma(n+1)}\left(\frac{A}{B}\right)^{2n}{}_2F_1\left[-n,-n;2;\left(\frac{B}{A}\right)^2\right]$ or, in integral form, $\frac{2D}{\pi B}\int_{-\infty}^{\infty}\frac{\cos\delta u}{u}J_0(Au)J_1(Bu)\,du$
 4. Ideal relay	$K(k)$ and $E(k)$ are the elliptic integrals of first and second kind, respectively. See Fig. D.2 and Sec. 5.1	$\frac{8D}{\pi^2 B}\frac{A}{B}\left\{E\left(\frac{B}{A}\right)-\left[1-\left(\frac{B}{A}\right)^2\right]K\left(\frac{B}{A}\right)\right\}$ for $\frac{B}{A}<1$ $\frac{8D}{\pi^2 B}E\left(\frac{A}{B}\right)$ for $\frac{B}{A}>1$
 7. Limiter	See Gibson and Sridhar (*op. cit.*) See Fig. D.3	$\frac{2D}{\pi\delta}\sum_{m=0}^{\infty}\sin\left(\frac{2D}{\delta}\sin^{-1}\frac{\delta}{B}\right)\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma(m+n)}{n!\,(m-n)!\,\Gamma(n+1)}\left(\frac{A}{B}\right)^{2n}{}_2F_1\left[-n,-n;2;\left(\frac{B}{A}\right)^2\right]$ or, in integral form, $\frac{2D}{\pi\delta B}\int_{-\infty}^{\infty}\frac{\sin\delta u}{u^2}J_0(Au)J_1(Bu)\,du$

$y = x^3$		$\frac{3}{4}B^2 + \frac{3}{2}A^2$
18. Cubic characteristic	See Fig. D.4 and Sec. 5.1	
$y = M \sin mx$	J_0 and J_1 are the Bessel functions of orders 0 and 1, respectively.	$\frac{2MJ_0(mA)J_1(mB)}{B}$
29. Harmonic nonlinearity	See Fig. D.5	

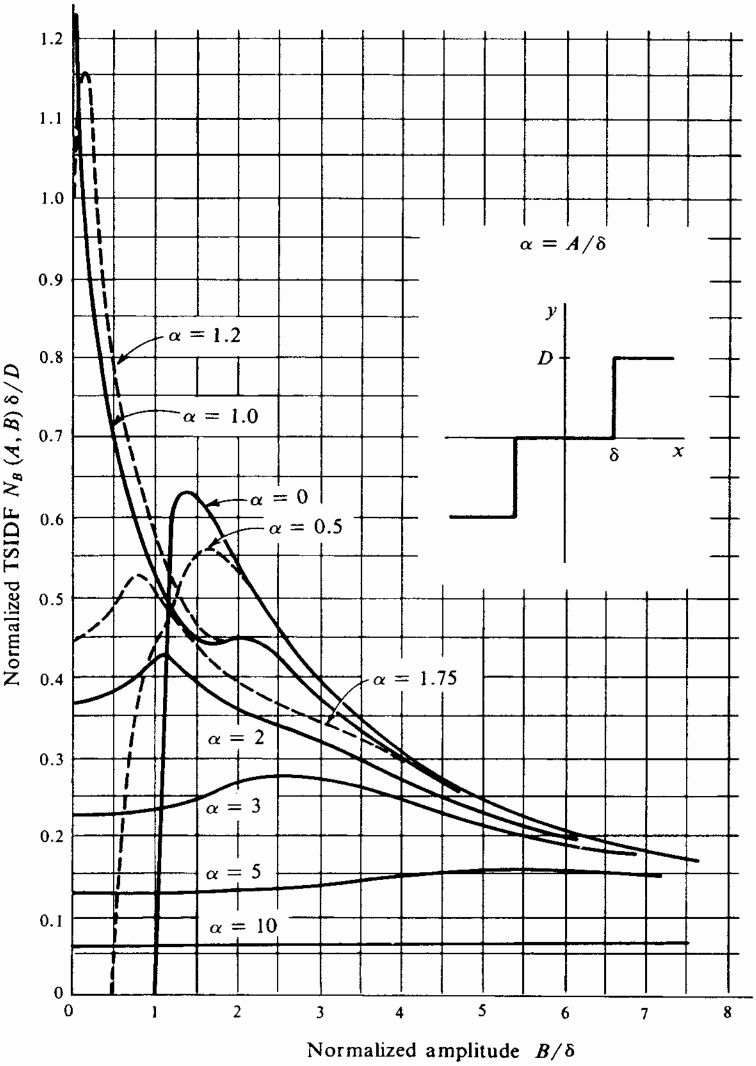


Figure D.1 *Normalized relay with dead zone TSIDF. (Gibson and Sridhar, Ref. 10 of Chap. 5.)*

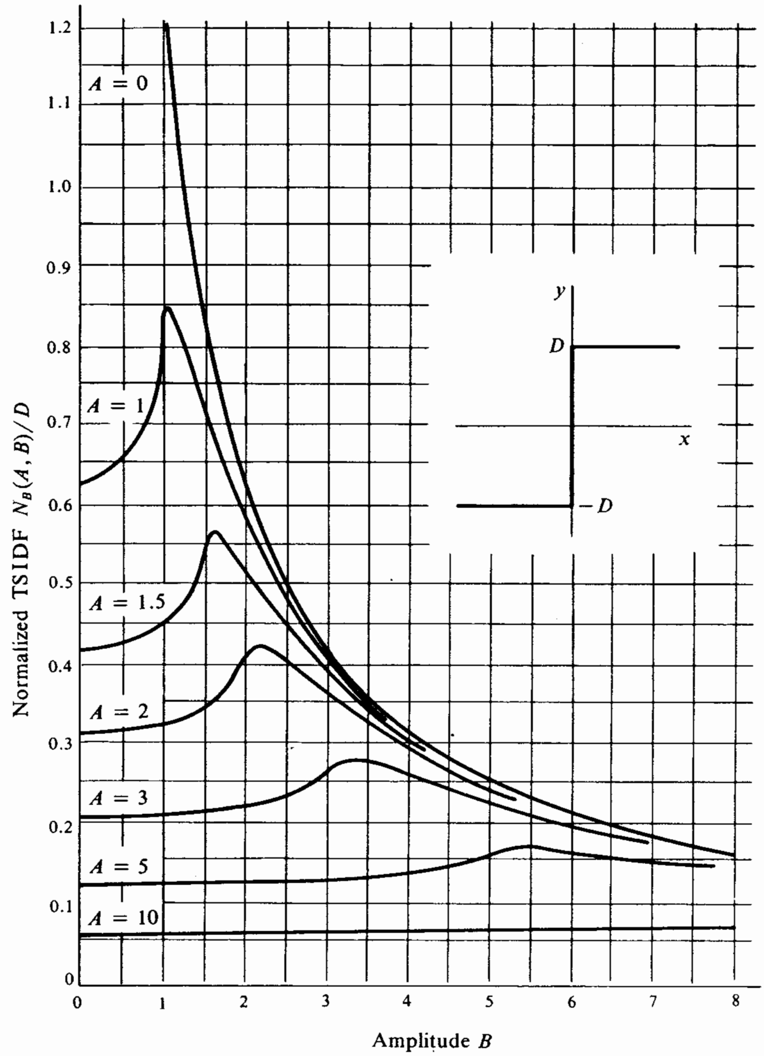


Figure D.2 Normalized ideal-relay TSIDF.

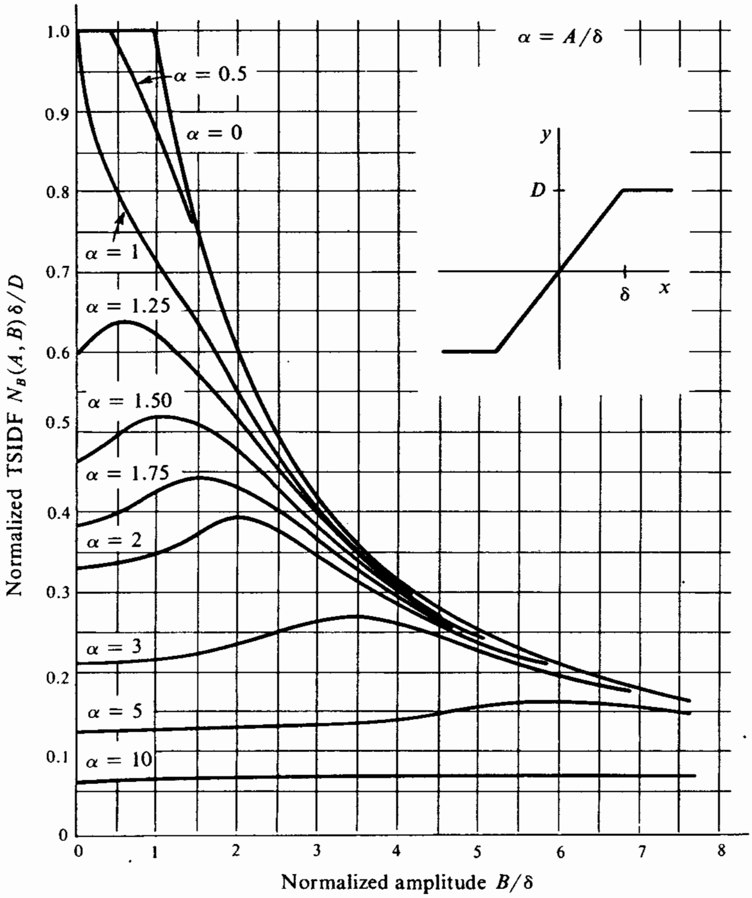


Figure D.3 *Normalized limiter TSIDF.* (*Gibson and Sridhar, Ref. 10 of Chap. 5.*)

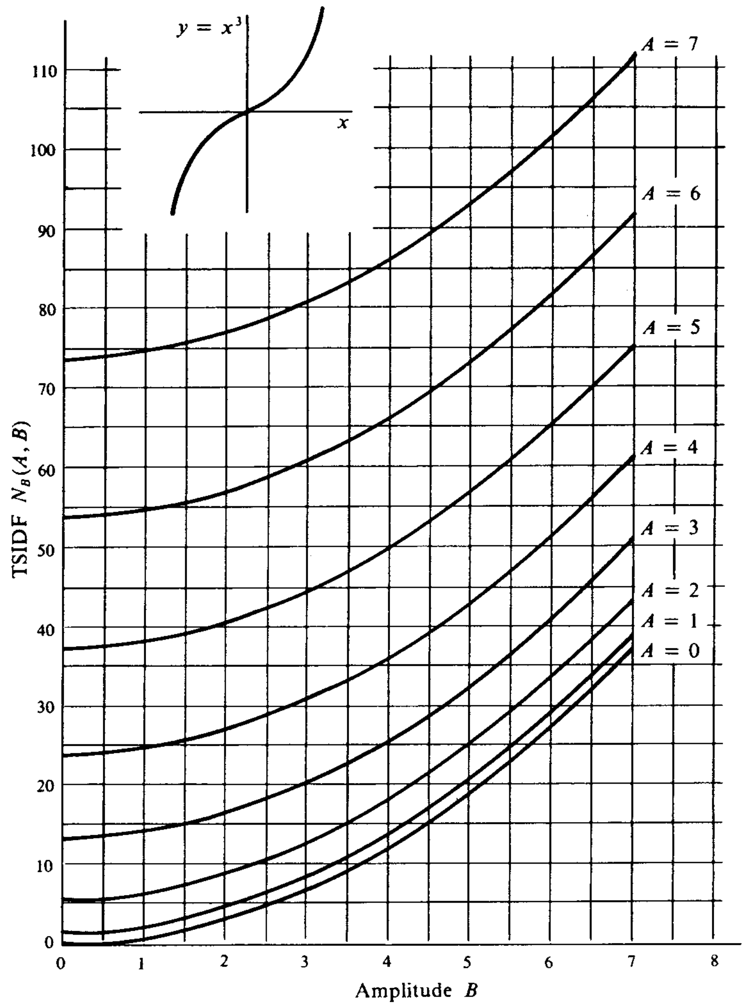


Figure D.4 Cubic characteristic TSIDF.

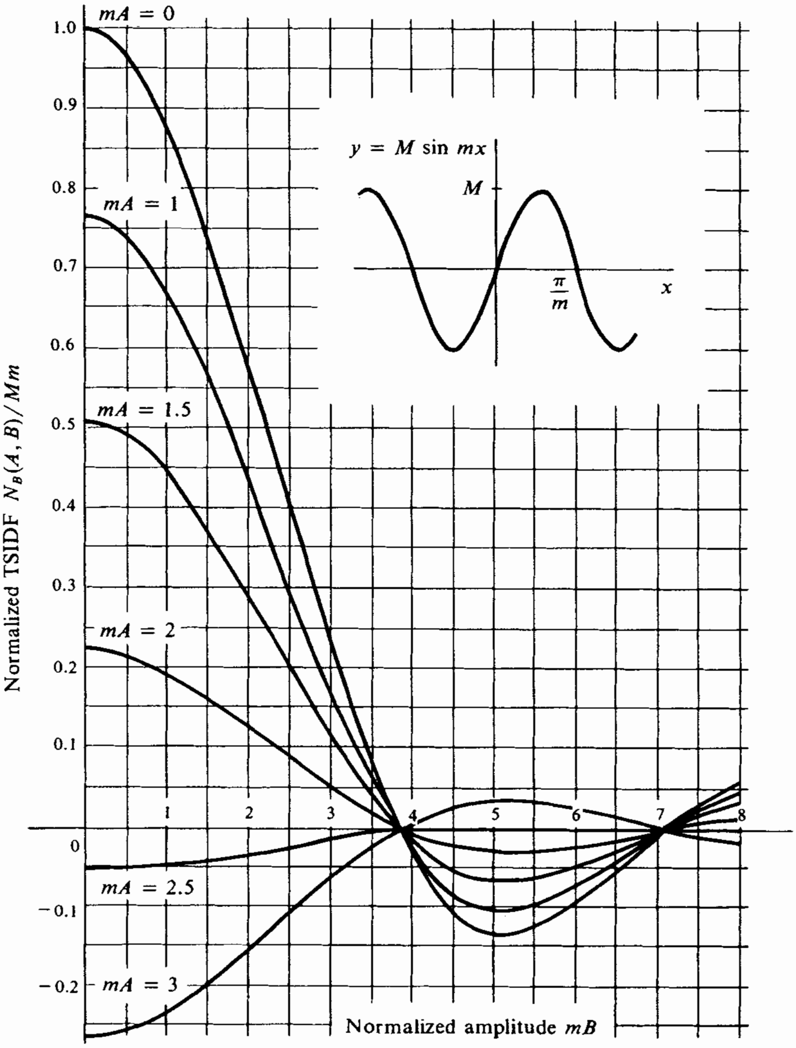


Figure D.5 Normalized harmonic nonlinearity TSIDF.