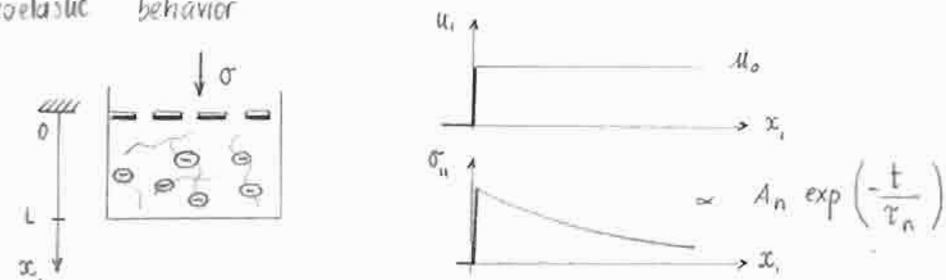
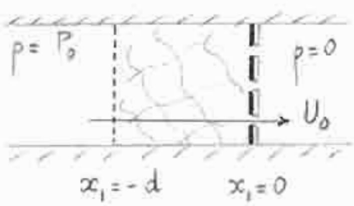
L # 17 by Prof. Alan Grodzinsky

Today: electromechanical properties of biological tissues
- I. electrokinetic transduction in tissues, gels, networks
  - A examples: streaming potential, electroosmosis
  - B. molecular origin of fixed charge groups
- II. tissue swelling & molecular repulsion
- III. length scales: tissues (mm) → cells MEMS (μm) → molecules (nm)

• Poroelastic behavior



problem 6.1



find $u_1(x_1)$

$\varepsilon_{11}(x_1)$

problem 6.2

• The equations are

(1) $\sigma_{11} = \underbrace{(2G + \lambda)}_{H} \varepsilon_{11} - p$ — stress-strain constitutive law

(2) $U_1 = -\dfrac{\partial u_1}{\partial t} + U_0$ — conservation of mass

(3) $\dfrac{\partial \sigma_{11}}{\partial x_1} = 0$ — conservation of momentum

(4) $U_1 = -k_{11}\dfrac{\partial P}{\partial x_1} + k_{12}\dfrac{\partial V}{\partial x_1}$ — fluid / solid interactions, Darcy's law

(5) $J_1 = k_{21}\dfrac{\partial P}{\partial x_1} - k_{22}\dfrac{\partial V}{\partial x_1}$ — Ohm's law (field version)

(6) $\dfrac{\partial J_1}{\partial x_1} = 0$ — conservation of charge

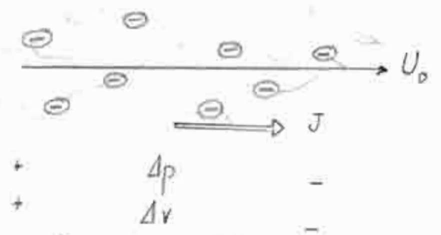
• With the help of (1) to (6), we'll explain why electric fields induce fluid flow / fluid flows induce electric field.

This effect is different from the specific piezo-electric effect (quartz-related) is due to friction interaction between water and positively charged ions, which tend to move along the electric field (the negatively charged solid phase can't move). Electrophoresis uses the same phenomenon (this time, negatively charged particles move with respect to the positively charged fluid phase)

• k_{12} and k_{21} show this electrical / mechanical coupling

reciprocal system: $k_{21} = k_{12}$ (electroosmosis = electrophoresis)

- Electrokinetic constitutive laws



empirical → complete equation (4)

- $U\big|_{\frac{\partial p}{\partial x}=0} = k_{12}\dfrac{\partial V}{\partial x_1}$ where k_{12} electroosmotic coefficient

It can be shown ("chapter 9 handout") that

$k_{12} = k_{11}\,\rho_m$

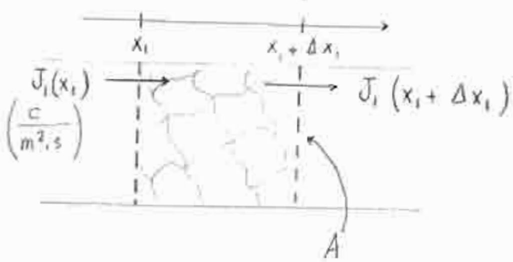
↳ fixed charge density of tissue $\left(\frac{C}{m^3}\right)$

Electric field $(V.m^{-1})$ $E_1 = -\dfrac{\partial V}{\partial x_1}$

J has the units of $A\ m^{-2}$ or $C.\ m^{-2}.\ s^{-1}$

$\bar{J}_1 = \sigma\ \bar{E}_1$ with σ in $mho.m^{-1}$ $(\sigma = k_{22})$

- Conservation of charge



flux of charge into the element in Δt = rate of accumulation of charge

$$A\left[J_1(x_1) - J_1(x_1+\Delta x_1)\right]\Delta t = A\ \rho_e\ \Delta x_1$$

↳ $\frac{C}{m^3}$ $\rho_e = \rho_m + \rho_{ions}$

divide by $A\cdot\Delta x_1\cdot\Delta t$:

$$\boxed{\frac{\partial J_1}{\partial x_1} = -\frac{\partial \rho_e}{\partial t}}$$

conservation of charge
no net charge in volume even if streaming potential after $\tau \sim 1$ ns

charge relaxation
for $t >$ few ns, $\bar{\nabla}\cdot\bar{J} = 0$

- Summary: new constitutive law ((4) and (5))

$$\begin{bmatrix}\bar{U}\\ \bar{J}\end{bmatrix} = \begin{bmatrix}-k_{11} & k_{12}\\ k_{21} & -k_{22}\end{bmatrix}\cdot\begin{bmatrix}\nabla p\\ \nabla V\end{bmatrix}$$

now, the permeability depends on k_{11}, k_{22}, k_{21}, k_{12}
open- and short-circuit permeabilities are different

Combining (1) to (6), we get a new diffusion equation

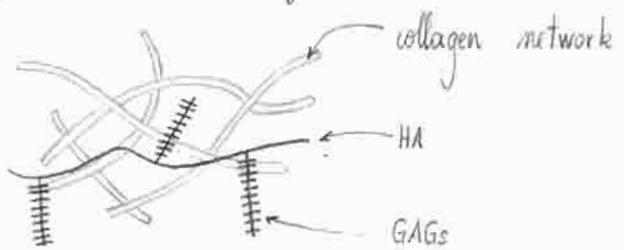
$$\boxed{\frac{\partial u_1}{\partial t} = H\,\text{"}k\text{"}\,\frac{\partial^2 u_1}{\partial x_1^2} + U_0 + \frac{k_{12}}{k_{22}}J_0}$$

U_0, J_0: drives

$\text{"}k\text{"} = k_{11} - \dfrac{k_{12}\,k_{21}}{k_{22}}$ open circuit

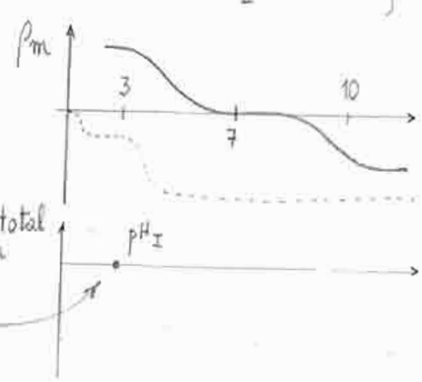
"k" is always > 0. the electric field reduces the permeability
doesn't change the direction of the flow

- Origin of fixed charge



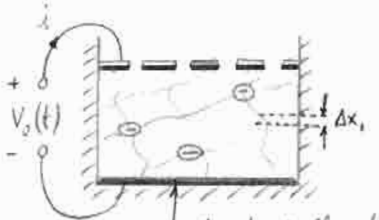
$SO_3H \rightleftharpoons SO_3^- + H^+$
$COOH \rightleftharpoons COO^- + H^+$
$NH_3^+ \rightleftharpoons NH_2 + H^+$

in GAGs: SO_3^- and COO^-
net charge depends on pH



ρ_m: matrix charge density
— collagen
--- aggrecan
— cartilage (total)

isoelectric pH = no net charge



find ① $\sigma_{11}(t)$ caused by a step compression
② streaming potential $V_0(t)$
relaxation times are the same , even if initial time behaviors are not.

Ag / AgCl electrode

- from conservation of charge $\frac{\partial J_1}{\partial x_1} = 0 \Rightarrow J_1 =$ constant (charge relaxation is so fast)
- use electrometer amplifier $i = 0 \Rightarrow J_1 = 0$ (instrument doesn't allow the passage of current, "open circuit voltage")
- from (5) with $J_1 = 0$

$$\boxed{\frac{\partial V}{\partial x_1} = \frac{k_{21}}{k_{22}} \cdot \frac{\partial p}{\partial x_1}}$$

functions of position
(sum of $H\varepsilon_{11}$ and pressure $-p$ constant with x_1 but each contribution changes with x_1)

- use (1) through (4) to find $\frac{\partial p}{\partial x_1}$, and hence $\int_L^0 \frac{\partial V}{\partial x_1}\, dx_1 = V(0) - V(x_1 = L) \equiv V_0$

in terms of $\frac{k_{21}}{k_{22}}$ streaming potential coefficient

The phase angle of this streaming potential is different for fluid (here) cause.
solid (piezoelectric)