5.60 Thermodynamics & Kinetics
Spring 2008

Math Review

Differentiation

$$df = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy \qquad \text{and if } z = z(x,y)$$

$$\left(\frac{\partial f}{\partial x}\right)_z = \left(\frac{\partial f}{\partial x}\right)_y + \left(\frac{\partial f}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_z$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

$$\left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_z \quad or \quad \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

If df=gdx+hdy is exact then $\left(\frac{\partial g}{\partial y}\right)_x = \left(\frac{\partial h}{\partial x}\right)_y$

Examples:

$$\frac{d}{dx}x^r = rx^{r-1}$$

$$\frac{d}{dx}e^{3x} = 3e^{3x}$$

$$\frac{d}{dx}e^{3x^2} = 6xe^{3x^2}$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}log_a(x) = \frac{1}{x\ln(a)}$$

Sum Rule:

$$\frac{d}{dx}(5f + 3g) = 5\frac{df}{dx} + 3\frac{dg}{dx}$$ f=f(x); g=g(x)

Product Rule:

$$\frac{d}{dx}fg = f\frac{dg}{dx} + g\frac{dg}{dx}$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$$

Integration

Most common integrals:

$$\int_a^b \frac{1}{x}dx = \ln\left(\frac{b}{a}\right)$$

$$\int_a^b \frac{1}{x^2}dx = -\frac{1}{x}\Big|_a^b = \frac{1}{a} - \frac{1}{b}$$

$$\int_a^b x\,dx = \frac{1}{2}x^2\Big|_a^b = \frac{1}{2(b^2 - a^2)}$$

$$\int_a^b e^{3x}dx = \frac{1}{3}e^{3x}\Big|_a^b = \left(\frac{1}{3(e)^{3b}} - e^{3a}\right)$$

Quadratic Equation: solving for x

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

→ Simplifications (kinetics, equilibrium problems)

The quadratic equation shows up in kinetics questions and equilibrium questions. Typically, it is not required to solve the quadratic equation IF X IS SMALL.

Example:

$$\frac{x^2}{(a - x)(b - x)} = d$$

If x is assumed to be small, then,

$$\frac{x^2}{(a)(b)} = d$$

If the answer is small for x, then it is ok. If x turns out to be pretty big (0.4) then use the quadratic formulae!

Taylor Expansions: when x is small (especially in kinetics questions!)

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \cdots, \qquad -\infty < x < \infty$$

$$\ln(x) = x - \frac{1}{2}x^{2} + \frac{1}{3}x^{3} - \frac{1}{4}x^{4} + \cdots$$ For $x < 1$

Partial Fractions

Example:

$$\frac{x-1}{(3x-5)(x-3)} = \frac{A}{3x-5} + \frac{B}{x-3}$$

Multiply by denominator everywhere:

$$\frac{(x-1)(3x-5)(x-3)}{(3x-5)(x-3)} = \frac{A(3x-5)(x-3)}{3x-5} + \frac{B(3x-5)(x-3)}{x-3}$$

Simplify:

x-1=A(x-3)+B(3x-5)

Solve by setting values for x:

Let x=3; 2=A(0)+B(4) → B= 0.5

Let x=0; -1=A(-3)+0.5(-5) → A= -0.5

Final Answer:

$$\frac{x-1}{(3x-5)(x-3)} = \frac{-0.5}{3x-5} + 0.\frac{5}{x-3} = \frac{-1}{2(3x-5)} + \frac{1}{2(x-3)}$$

Math Tricks:

$\ln a - lnb = \ln\left(\frac{a}{b}\right)$

*when you have to equations in terms of ln, subtract the equations from each other and use the trick

Example:

$ln(p_1) = -\frac{3000}{T} + 13.1$ $\quad$ $ln(p_2) = -\frac{4000}{T} + 16.4$ $\quad$ given $\frac{p_2}{p_1} = 1$ $\quad$ ***(what point is this?)*

To solve for T and you are given the ratio of the pressures, subtract the equations:

$$ln(p_1) = \frac{3000}{T} + 13.1$$

$$- \quad ln(p_2) = -\frac{4000}{T} + 16.4$$

$$ln\left(\frac{p_1}{p_2}\right) = -\frac{3000}{T} + 13.1 - \left(-\frac{4000}{T} + 16.4\right)$$

$$lnxy = lnx + lny$$

$$\delta\left(\frac{1}{p}\right) = -\frac{1}{p^2}\partial p$$

$$\frac{1}{p}\partial p = \partial lnp$$

$$e^a e^b = e^{a+b}$$