1.033/1.57

Mechanics of Material Systems

(Mechanics and Durability of Solids I)

Franz-Josef Ulm

If Mechanics was the answer, what was the question ?

- Traditional:
  – Structural Engineering
  – Geotechnics

Structural Design
- Service State (Elasticity)
- Failure (Plasticity or Fracture)
- Mechanism

If Mechanics was the answer, what was the question ?

- Material Sciences and Engineering
  - New materials for the Construction Industry

Micromechanical Design of a new generation of Engineered materials

Concrete with Strength of Steel

If Mechanics was the answer, what was the question ?

- Diagnosis and Prognosis – Anticipating the Future

If Mechanics was the answer, what was the question ?

- Diagnosis and Prognosis – Anticipating the Future

If Mechanics was the answer, what was the question ?

- Traditional:
  - Structural Engineering
  - Geotechnics
  - …
- Material Sciences and Engineering
  - New materials for the Construction Industry
  - Engineered Biomaterials,…
- Diagnosis and Prognosis – Anticipating the Future
  - Pathology of Materials and Structures (Infrastructure Durability, Bone Diseases, etc.)
  - Give numbers to decision makers…

If Mechanics was the answer, what was the question ?

- 1.033/1.57 – Fall 01
  Mechanics and Durability of Solids I:
  - Deformation and Strain
  - Stress and Stress States
  - Elasticity and Elasticity Bounds
  - Plasticity and Yield Design
- 1.570 – Spring 01
  Mechanics and Durability of Solids II:
  - Damage and Fracture
  - Chemo-Mechanics
  - Poro-Mechanics
  - Diffusion and Dissolution

Content 1.033/1.57

Part I. **Deformation and Strain**

1 Description of Finite Deformation

2 Infinitesimal Deformation

Part II. **Momentum Balance and Stresses**

3 Momentum Balance

4 Stress States / Failure Criterion

Part III. **Elasticity and Elasticity Bounds**

5 Thermoelasticity,

6 Variational Methods

Part IV. **Plasticity and Yield Design**

7 1D-Plasticity – An Energy Approac

8 Plasticity Models

9 Limit Analysis and Yield Design

Assignments 1.033/1.57

Part I. **Deformation and Strain**
- HW #1

Part II. **Momentum Balance and Stresses**
- HW #2
- Quiz #1

Part III. **Elasticity and Elasticity Bounds**
- HW #3
- Quiz #2

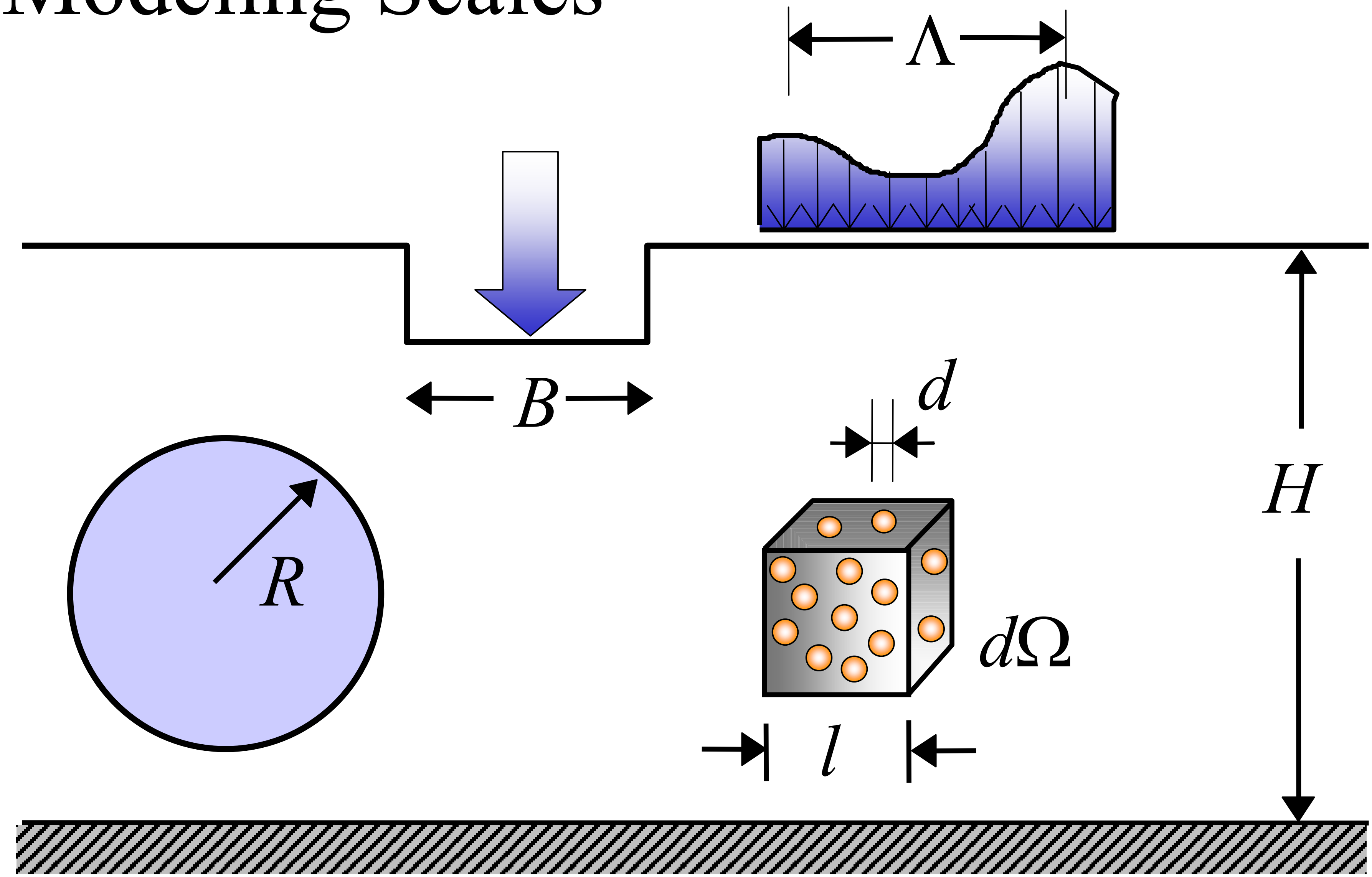
Part IV. **Plasticity and Yield Design**
- HW #4
- Quiz #3

FINAL

Part I: Deformation and Strain

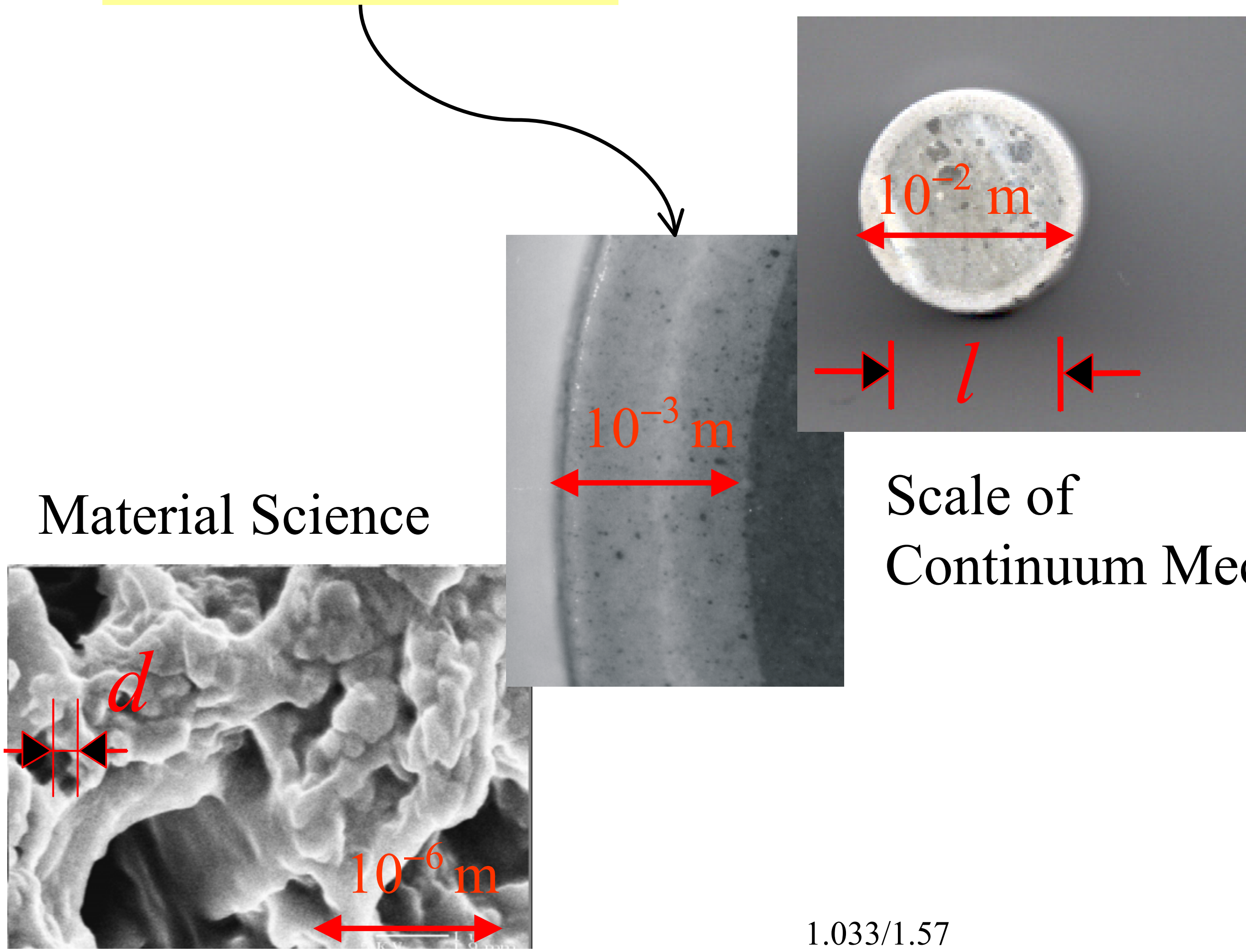
1. Finite Deformation

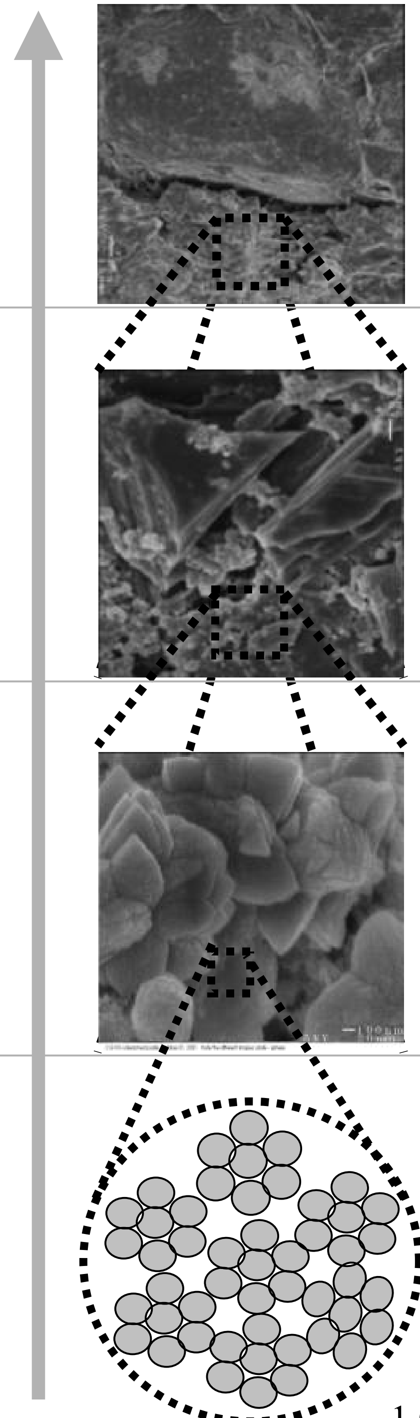
Modeling Scales



Modeling Scale (cont'd)

$$d << l << H$$





LEVEL III
Mortar,
Concrete
$> 10^{-3}$ m

Cement paste plus sand and Aggregates, eventually Interfacial Transition Zone

LEVEL II
Cement Paste
$< 10^{-4}$ m

C-S-H matrix plus clinker phases, CH crystals, and macroporosity

LEVEL I
C-S-H matrix
$< 10^{-6}$ m

Low Density and High Density C-S-H phases (incl. gel porosity)

LEVEL '0'
C-S-H solid
10^{-9}–10^{-10} m

C-S-H solid phase (globules incl. *intra*-globules nanoporosity) plus *inter*-globules gel porosity

LEVEL III
Deposition scale
$> 10^{-3}$ m

Scale of deposition layers
Visible texture.

LEVEL II ('Micro')
Flake aggregation
and inclusions
$10^{-5} – 10^{-4}$ m

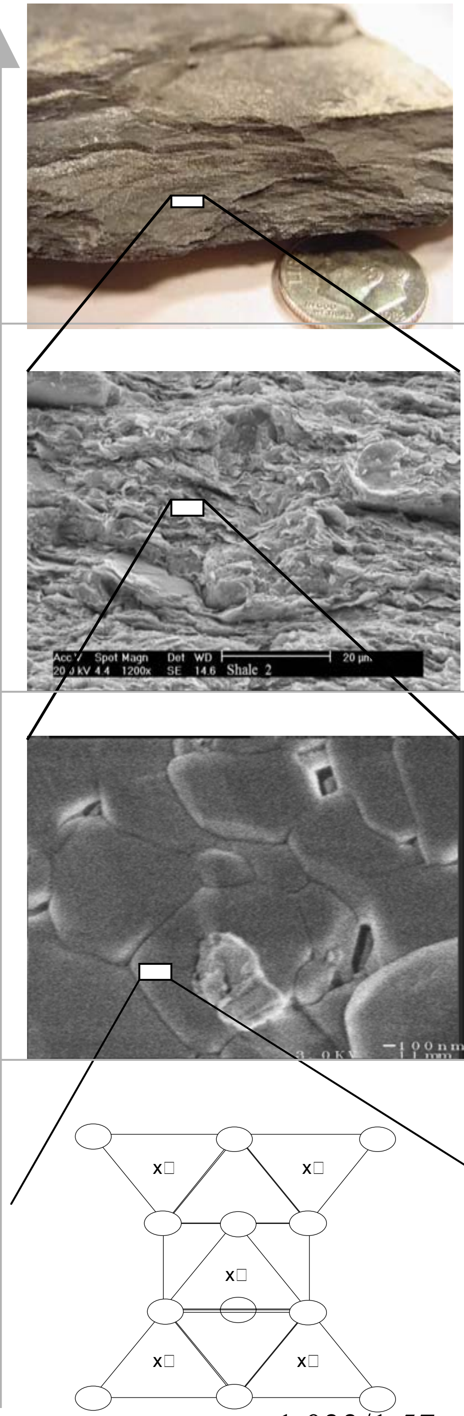
Flakes aggregate into layers,
Intermixed with silt size
(quartz) grains.

LEVEL I ('Nano')
Mineral
aggregation
$10^{-7} – 10^{-6}$ m

Different minerals aggregate
to form solid particles (flakes
which include nanoporosity).

LEVEL '0'
Clay Minerals
$10^{-9}–10^{-8}$ m

Elementary particles (Kaolinite,
Smectite, Illite, etc.), and
Nanoporosity (10 – 30 nm).



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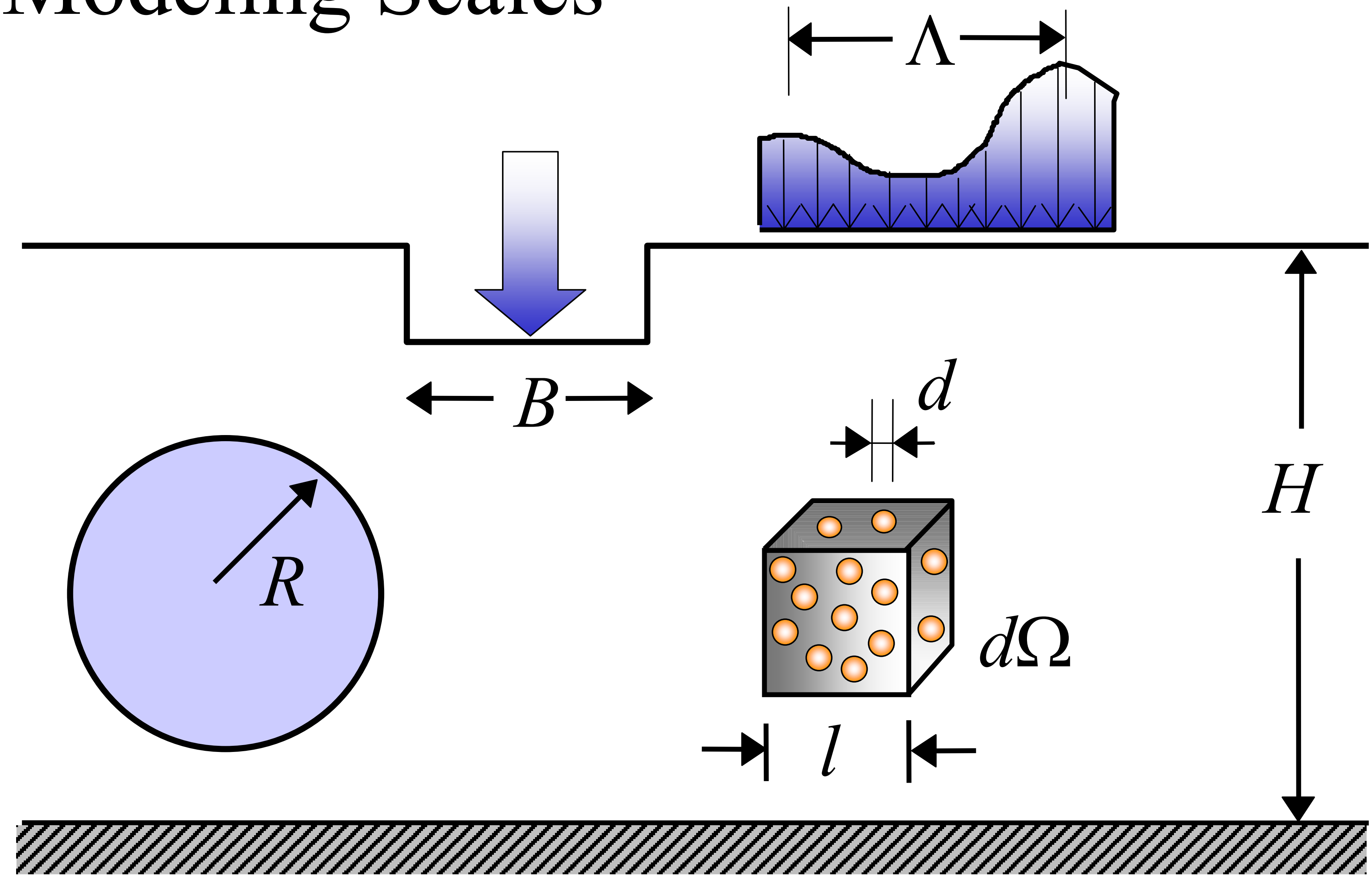
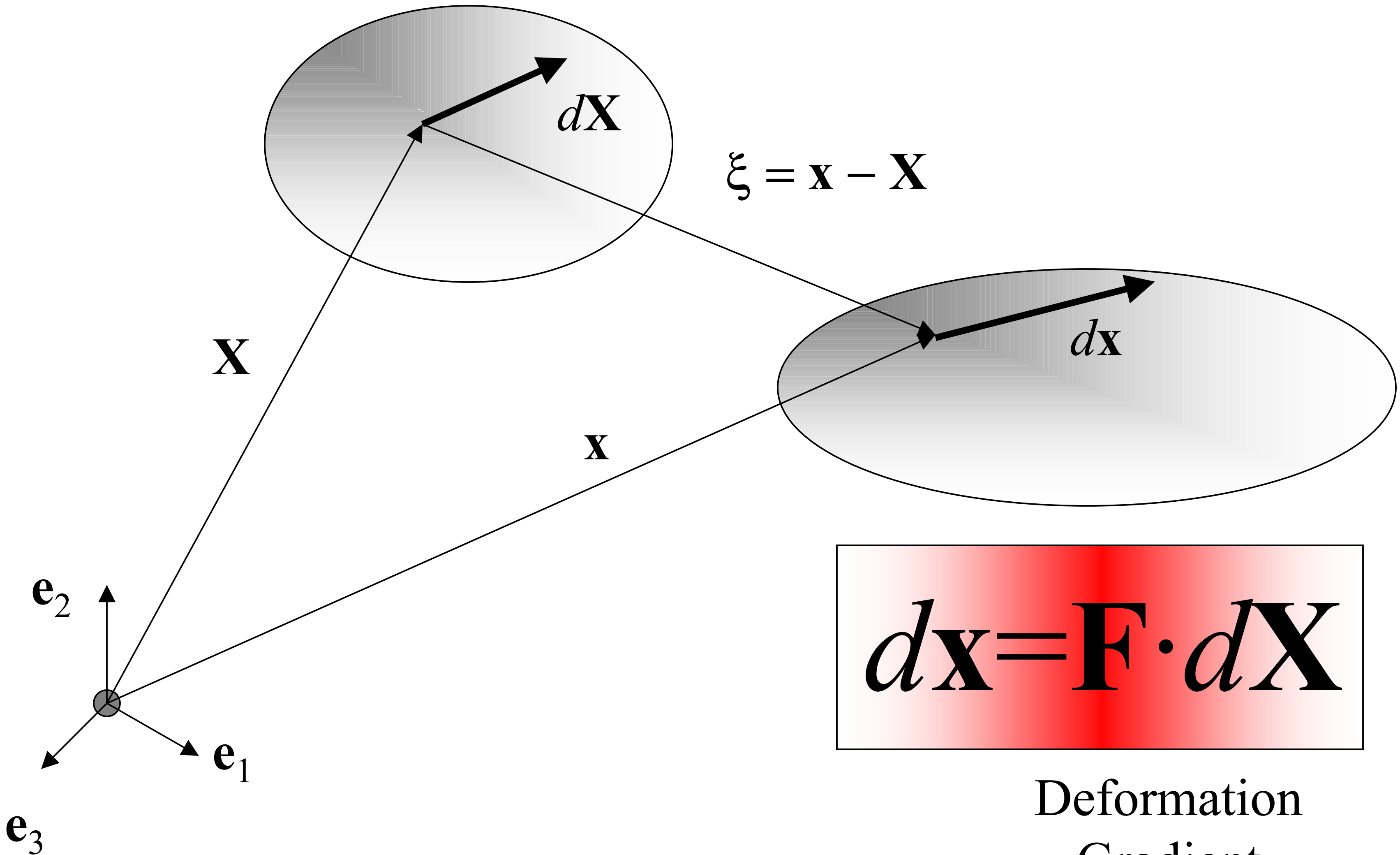
Modeling Scales



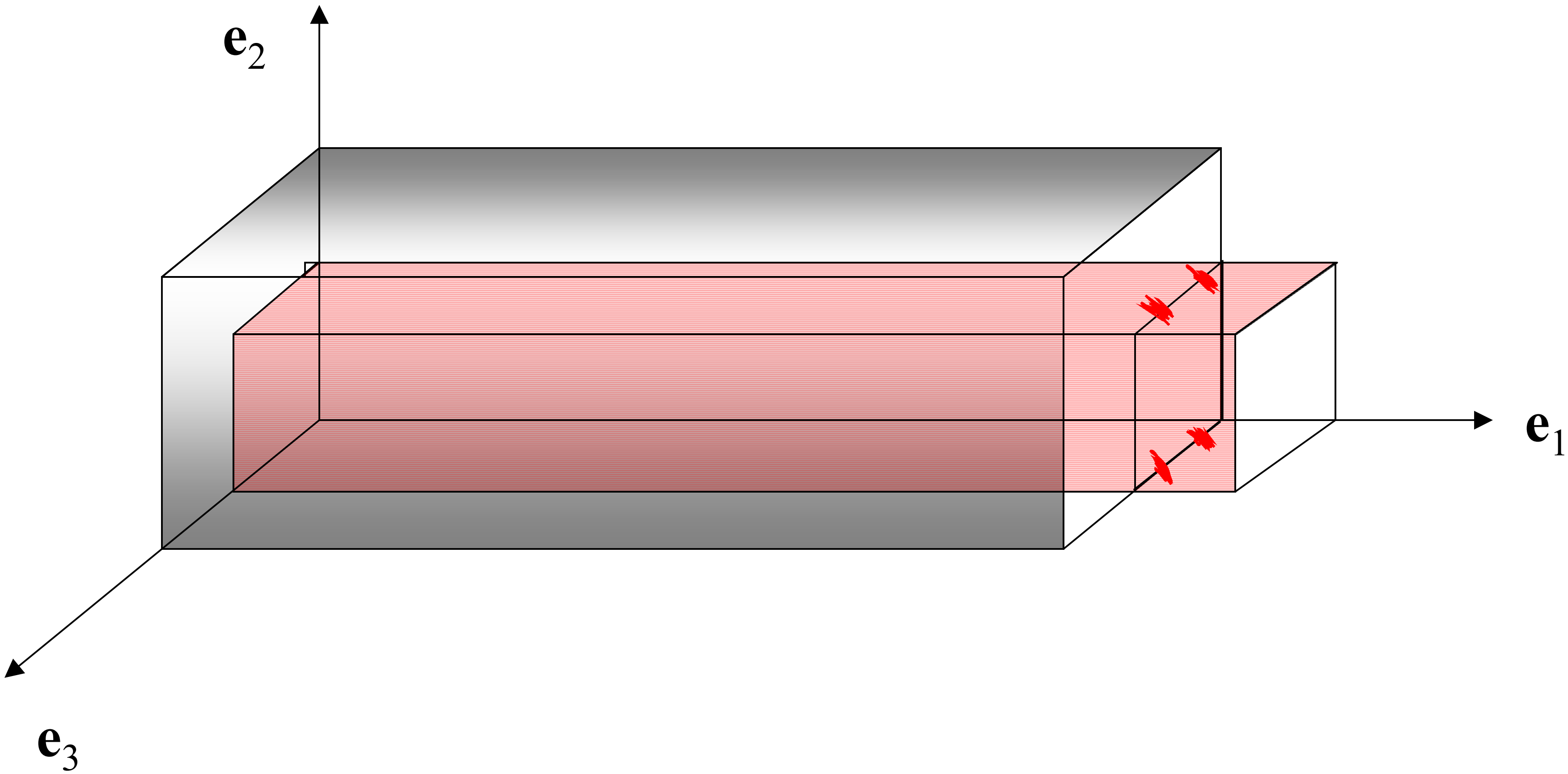
Transport of a Material Vector



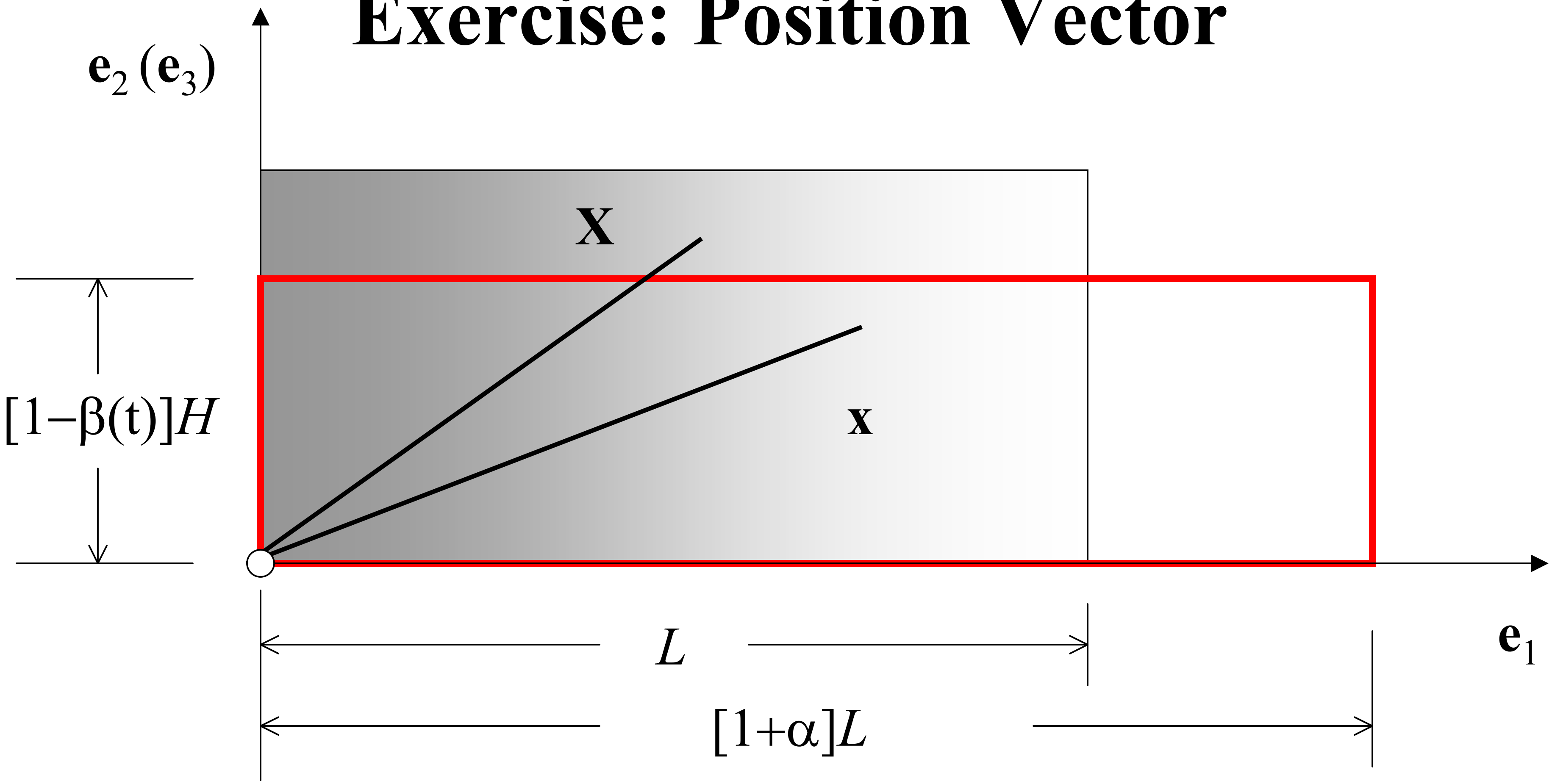
$$d\mathbf{x} = \mathbf{F} \cdot d\mathbf{X}$$

Deformation
Gradient

Exercise: Pure Extension Test

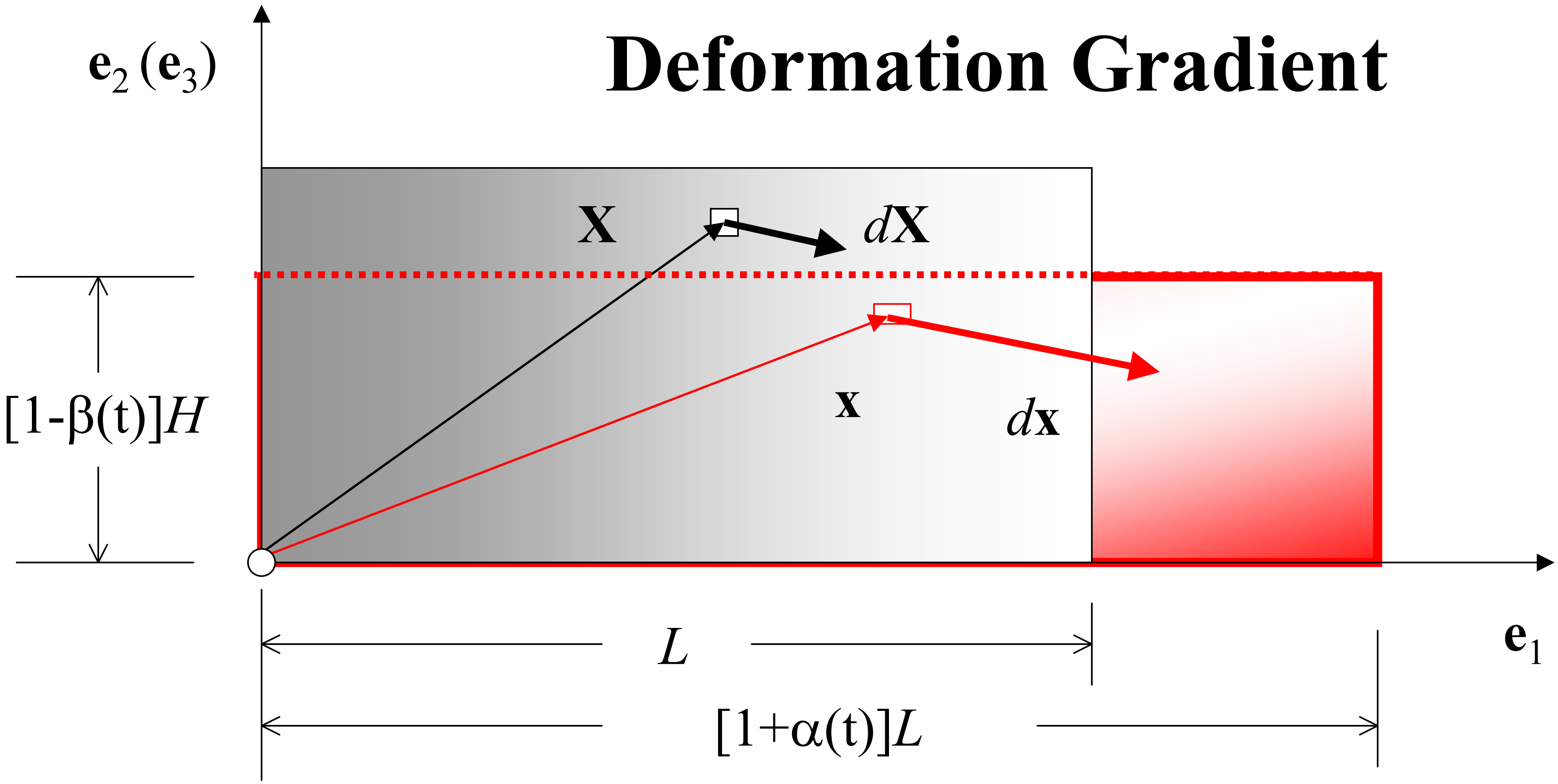


Exercise: Position Vector



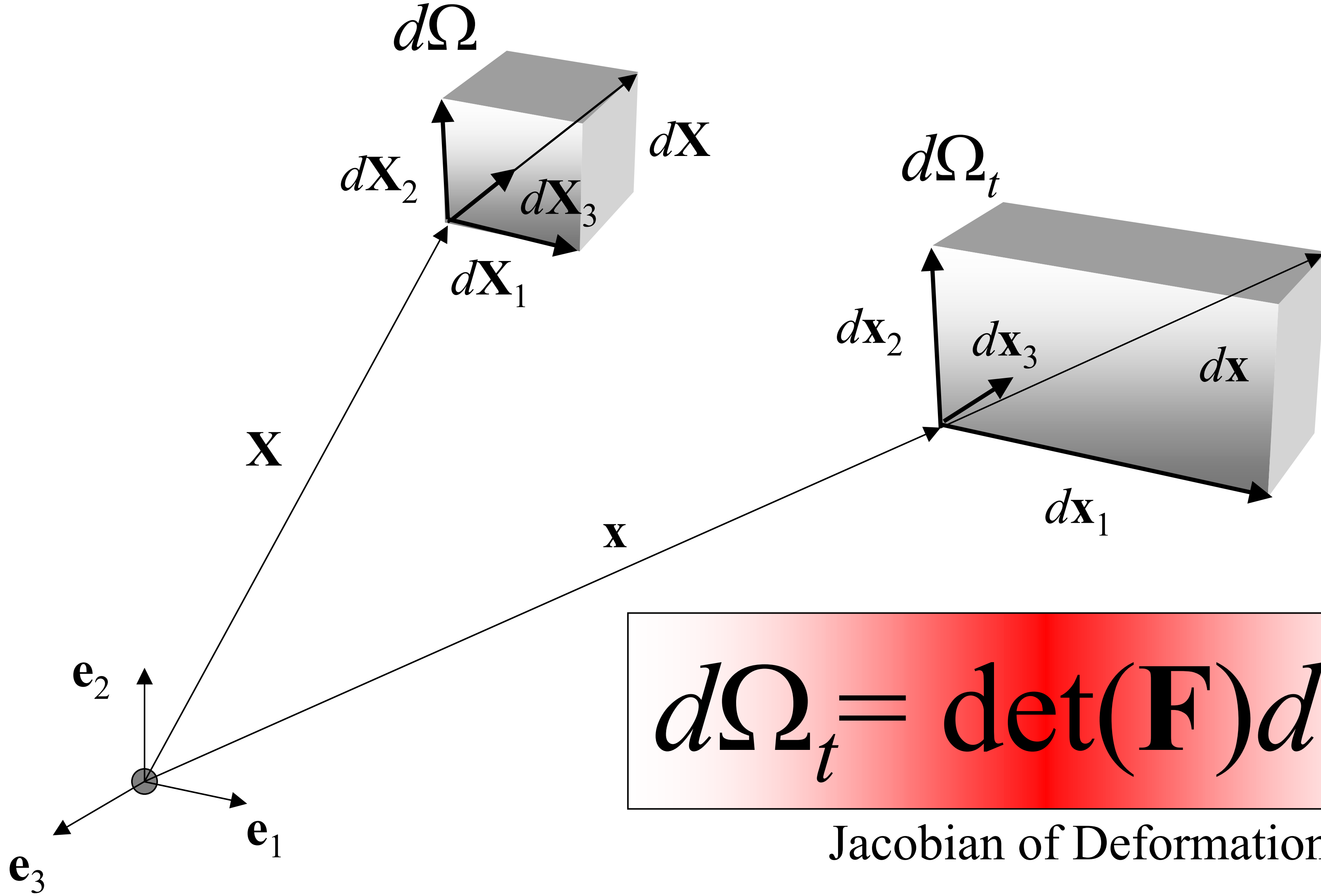
$x_1=X_1(1+\alpha)$; $x_2=X_2(1-\beta)$; $x_3=X_3(1-\beta)$;

Exercise: Material Vector / Deformation Gradient



$F_{11} = (1+\alpha)$; $F_{22} = F_{33} = (1-\beta)$

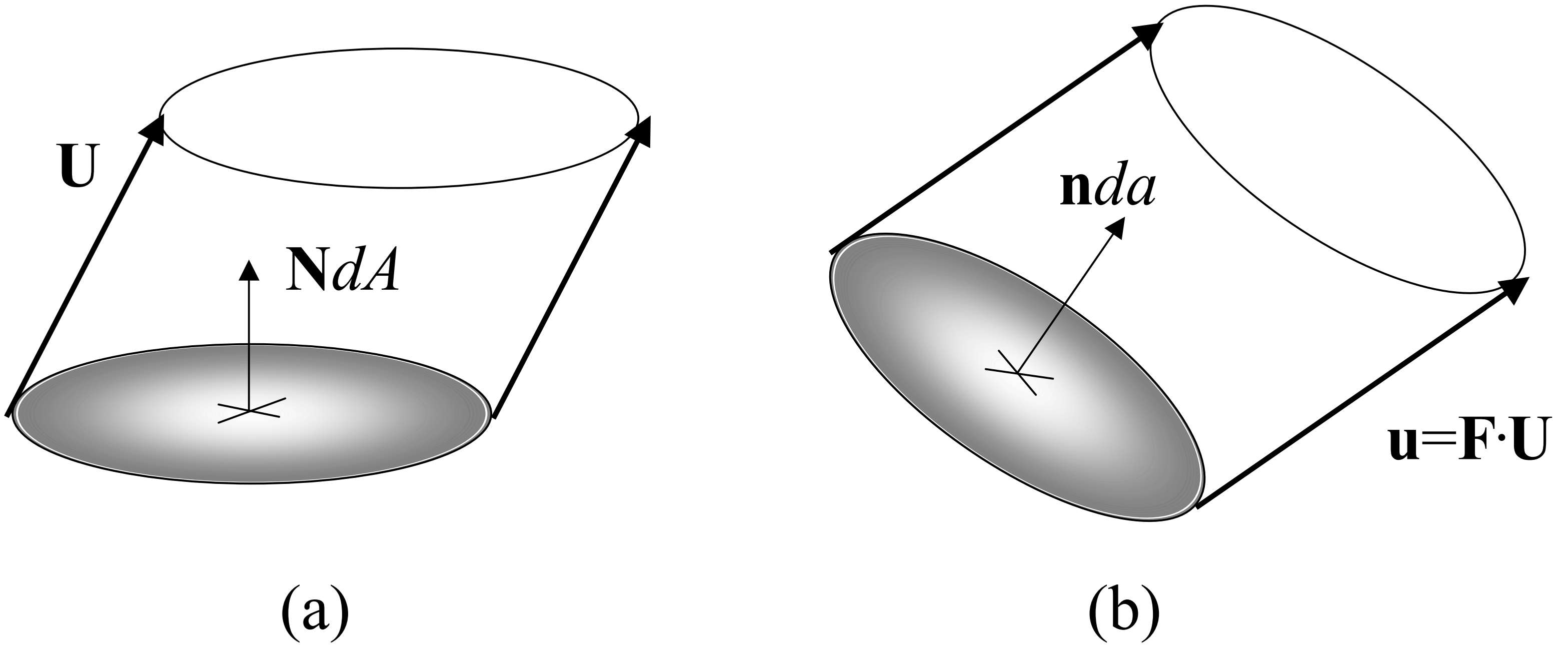
Volume Transport



$$d\Omega_t = \det(\mathbf{F})\, d\Omega$$

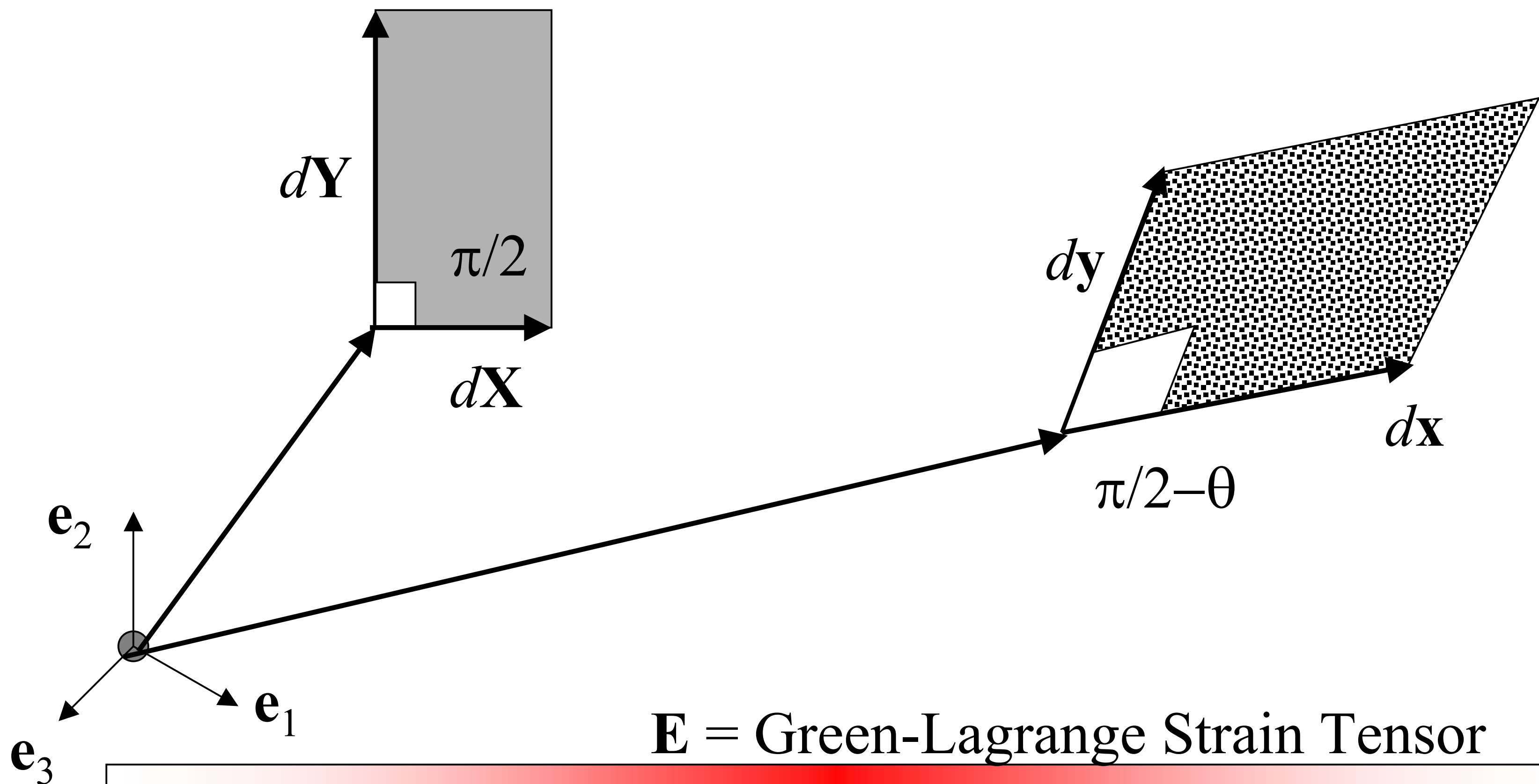
Jacobian of Deformation

Transport of an oriented material surface



$$\mathbf{n}\,da = J\,{}^{t}\mathbf{F}^{-1}\,\mathbf{N}\,dA$$

Transport of scalar product of two Material Vectors



E = Green-Lagrange Strain Tensor

$$d\mathbf{x}\cdot d\mathbf{y} = d\mathbf{X}\cdot(2\mathbf{E}+\mathbf{1})\cdot d\mathbf{Y}$$

Linear Dilatation and Distortion

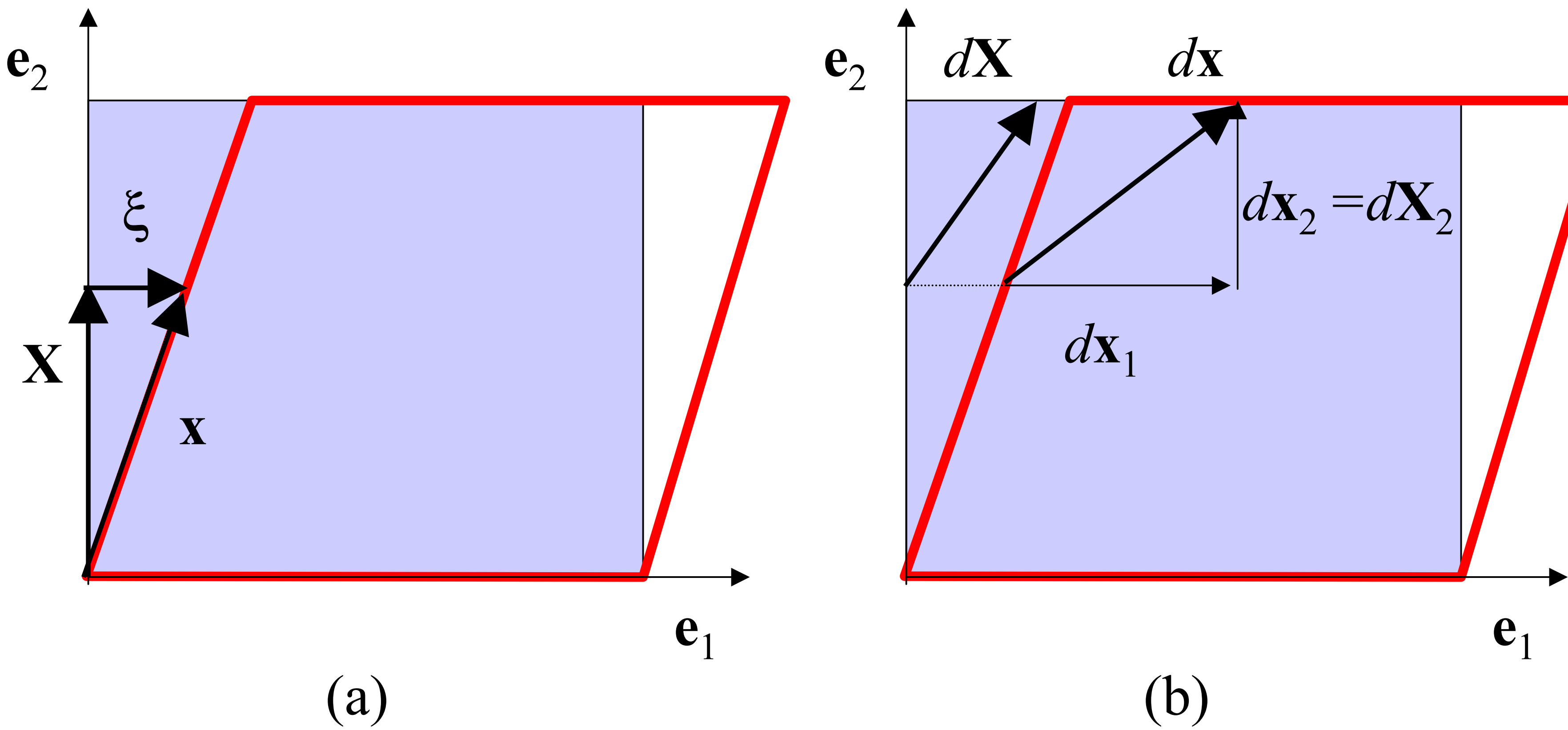
Length Variation of a Material Vector: Linear Dilatation

$$\lambda(\mathbf{e}_\alpha) = (1 + 2E_{\alpha\alpha})^{1/2} - 1$$

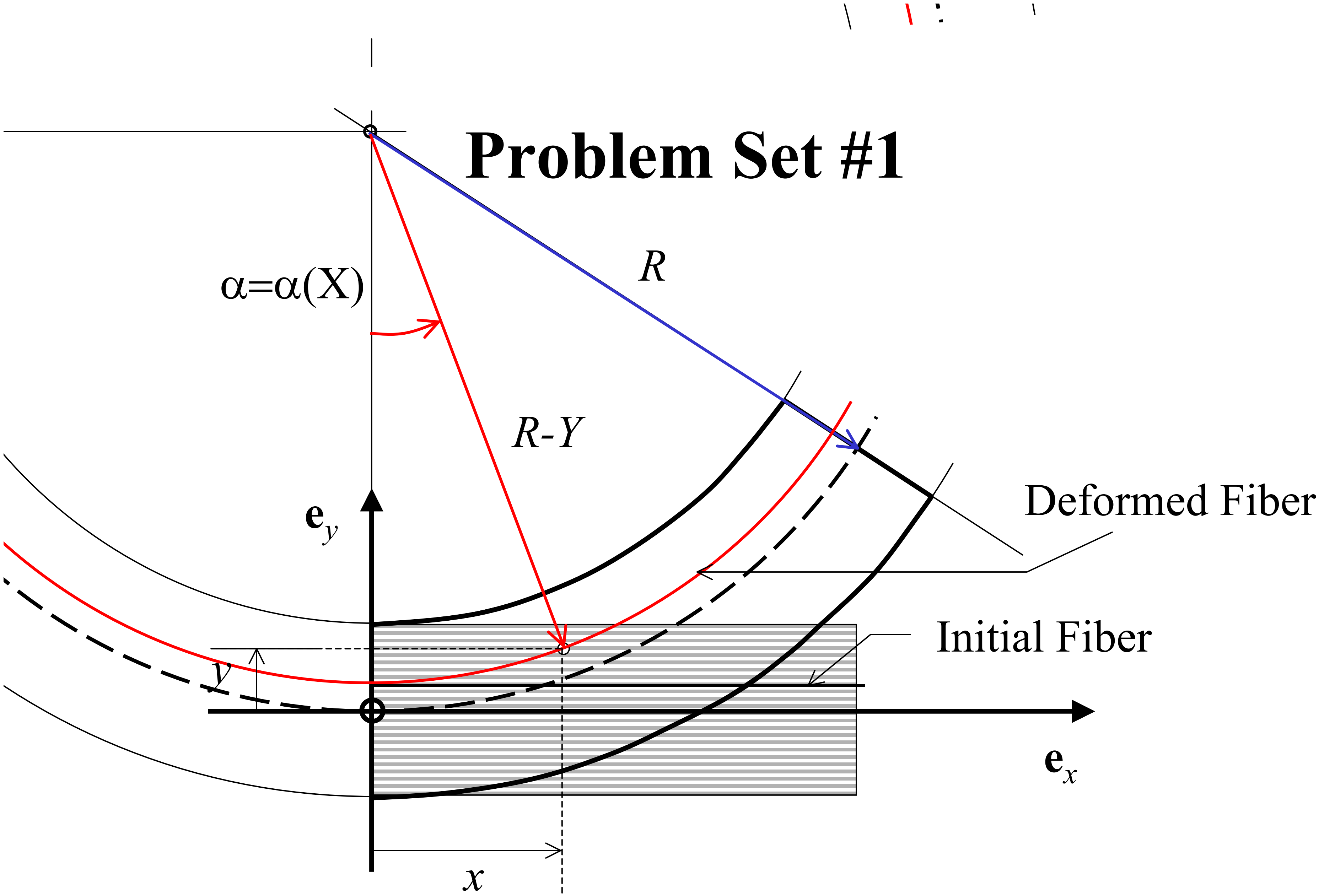
Angle Variation of two Material Vectors: Distortion

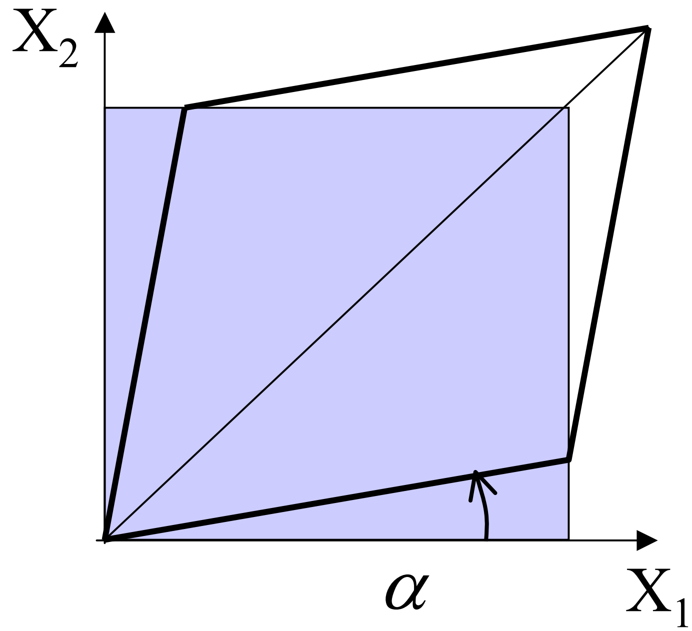
$$\sin\theta(\mathbf{e}_\alpha, \mathbf{e}_\beta) = \frac{2E_{\alpha\beta}}{[(1+2E_{\alpha\alpha})\,(1+2E_{\beta\beta})]^{1/2}}$$

Training Set: Simple Shear



Problem Set #1





double shear