1.033/1.57

Mechanics of Material Systems

(Mechanics and Durability of Solids I)

Franz-Josef Ulm

Lecture: MWF1 // Recitation: F 3:00-4:30

Part III: Elasticity and Elasticity Bounds

5. Thermoelasticity

Content 1.033/1.57

Part I. **Deformation and Strain**

1 Description of Finite Deformation

2 Infinitesimal Deformation

Part II. **Momentum Balance and Stresses**

3 Momentum Balance

4 Stress States / Failure Criterion

Part III. **Elasticity and Elasticity Bounds**

5 Thermoelasticity,

6 Variational Methods

Part IV. **Plasticity and Yield Design**

7 1D-Plasticity – An Energy Approac

8 Plasticity Models

9 Limit Analysis and Yield Design

The Necessity of Material Laws

- UNKNOWNS
  - **6** strains $\varepsilon_{ij} = \varepsilon_{ji}$
  - **6** stresses $\sigma_{ij} = \sigma_{ji}$
  - **3** displacements ξ_i

$\Sigma =$ **15**

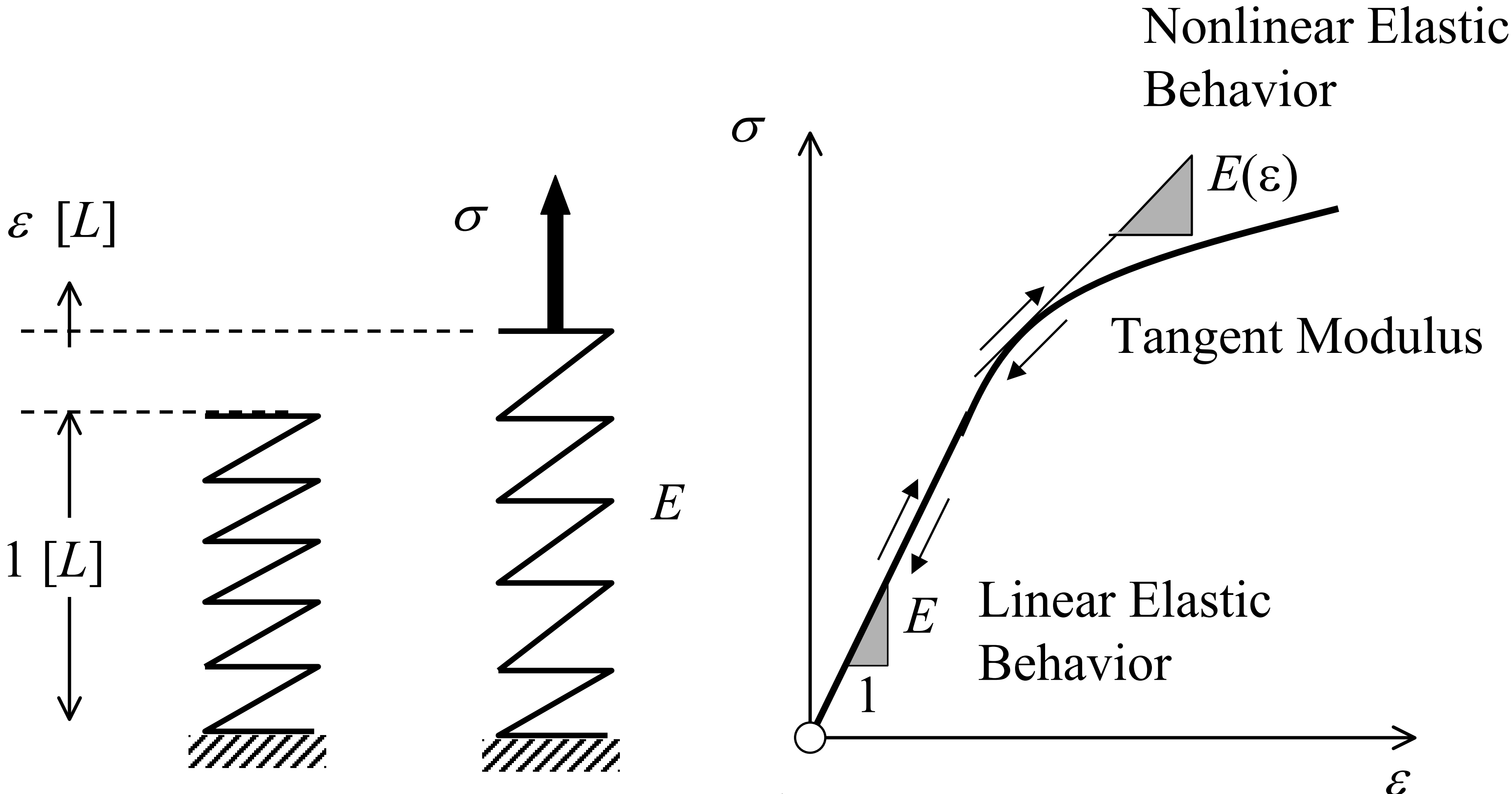
- EQUATIONS
  - **3** Momentum Balance $\sigma_{ij,j} + \rho f_i = 0$
  - **6** Strain-Displacement Relations: $2\varepsilon_{ij} = \xi_{i,j} + \xi_{j,i}$

$\Sigma =$ **9**

$\Delta = 15 - 9 =$ **6**

6 Missing Relations

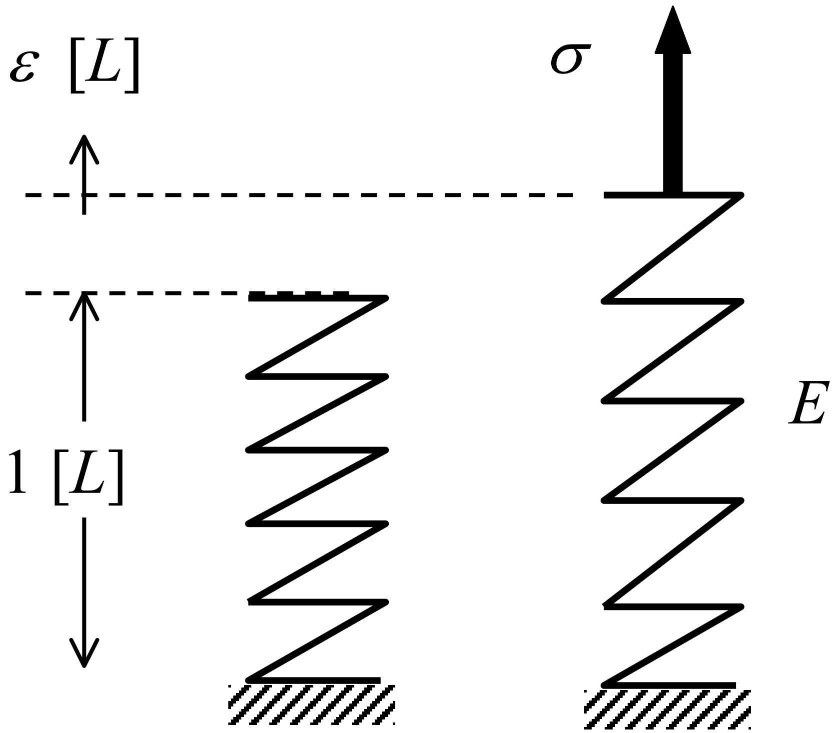
1D Think Model of Elasticity



Unique relation

$$\sigma \leftrightarrow \varepsilon; \quad \sigma = \sigma(\varepsilon); \quad \varepsilon = \varepsilon(\sigma)$$

Elasticity Potential



- $\underline{1^{st} + 2^{nd}\text{ Law}}$:

$$\varphi dt = \sigma\, d\varepsilon - d\psi = 0$$

Work — Stored Energy = Helmholtz Energy

- <u>1D-Model</u>:

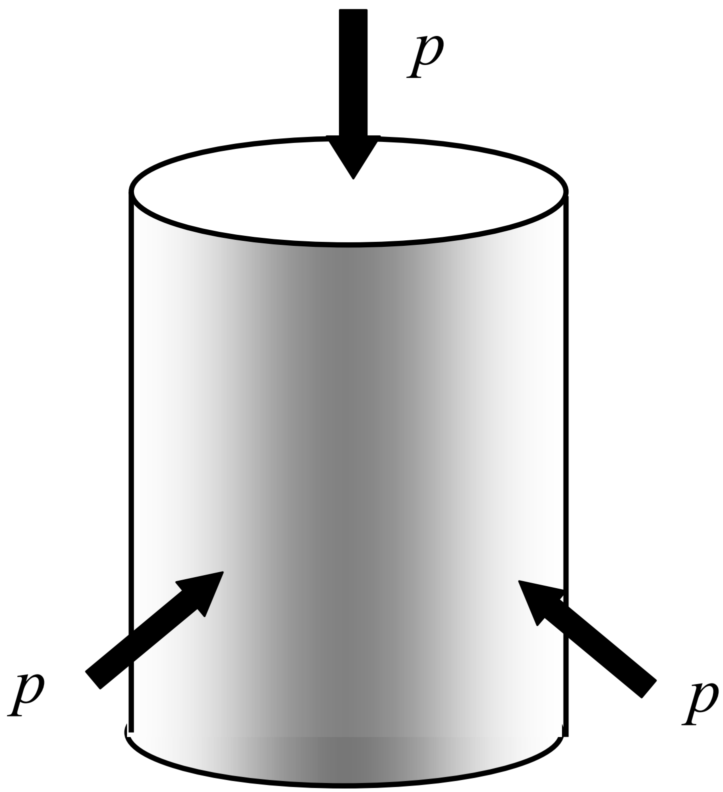
$$\psi = 1/2 E\varepsilon^2 \rightarrow \sigma = \frac{\partial \psi}{\partial \varepsilon}$$

- <u>3D-Model</u>:

$$\psi = \psi(\varepsilon_{ij}) \rightarrow \sigma_{ij} = \frac{\partial \psi}{\partial \varepsilon_{ij}}$$

Isotropic Elastic Material Properties

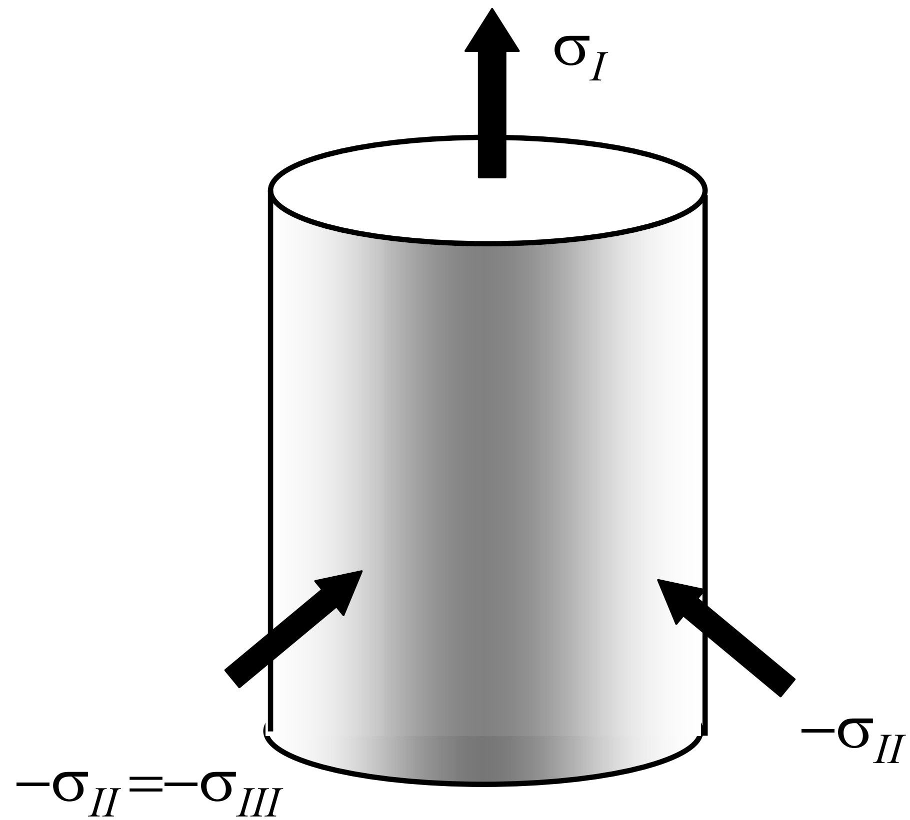
Hydrostatic Test



$$\sigma_m = -p = K\,\Delta V/V$$

Bulk Modulus

Triaxial Test

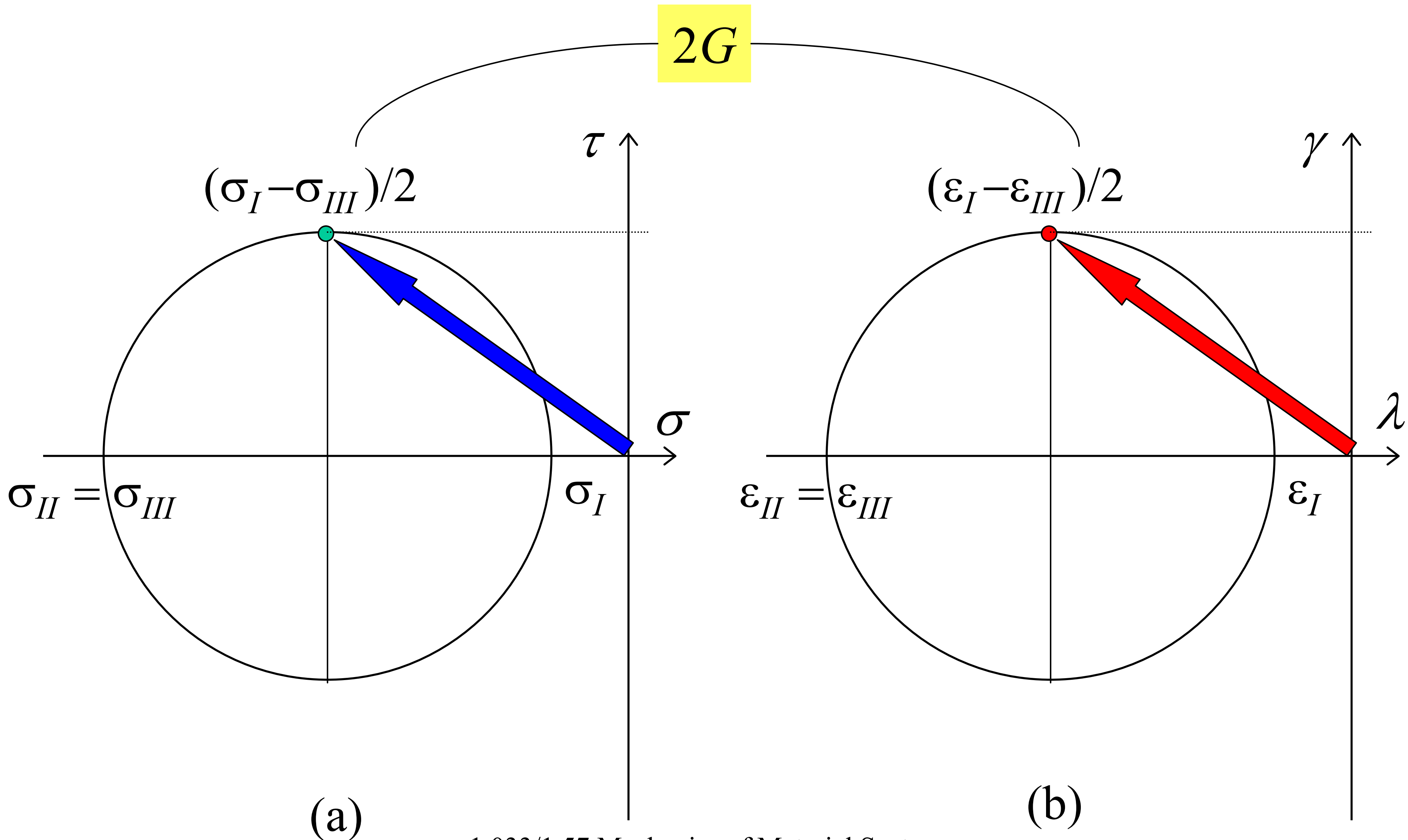


$$\sigma_I - \sigma_{III} = 2G\,(\varepsilon_I - \varepsilon_{III})$$

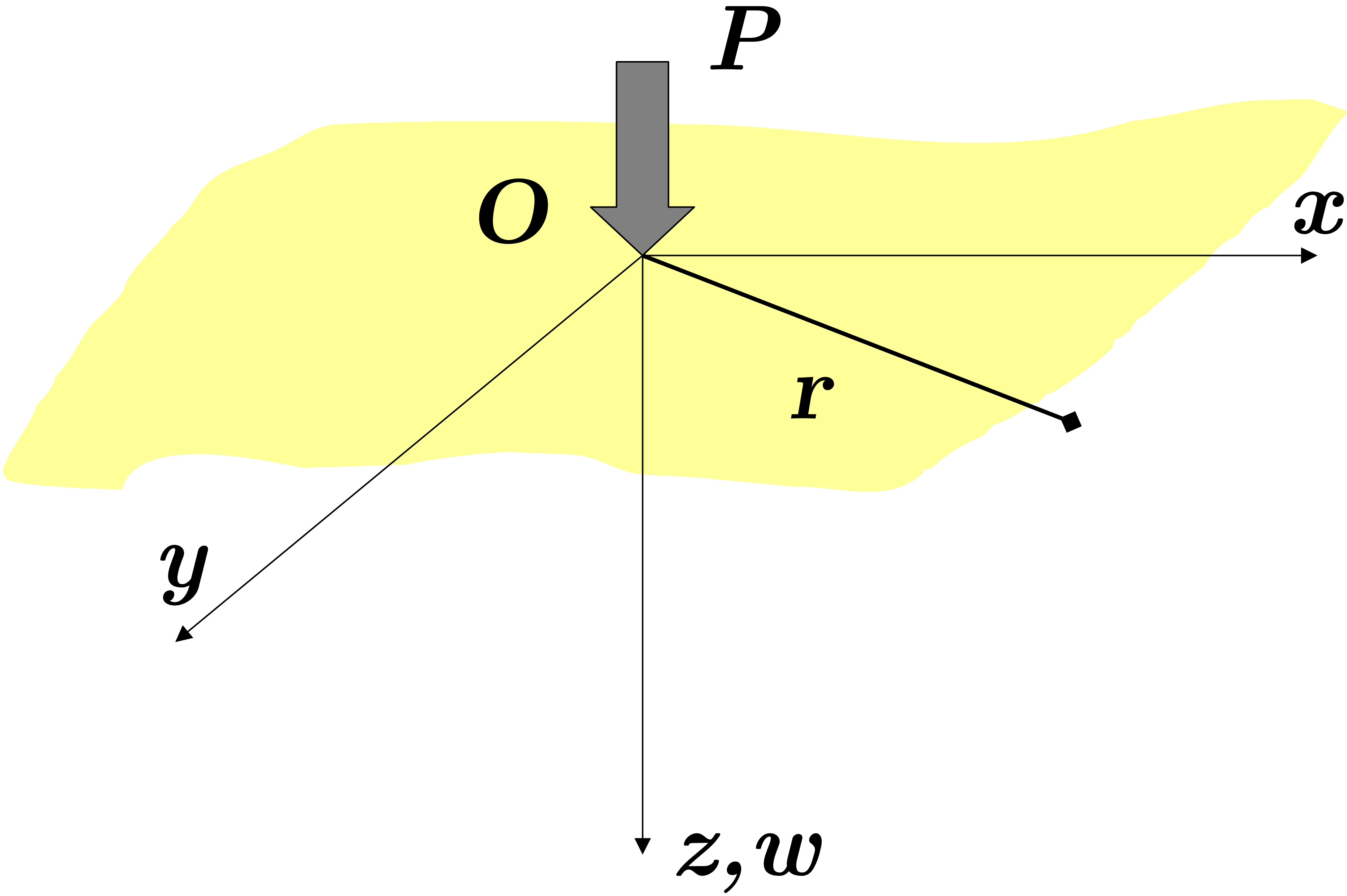
Shear Modulus

Shear Modulus – Triaxial Test

Mohr Representation

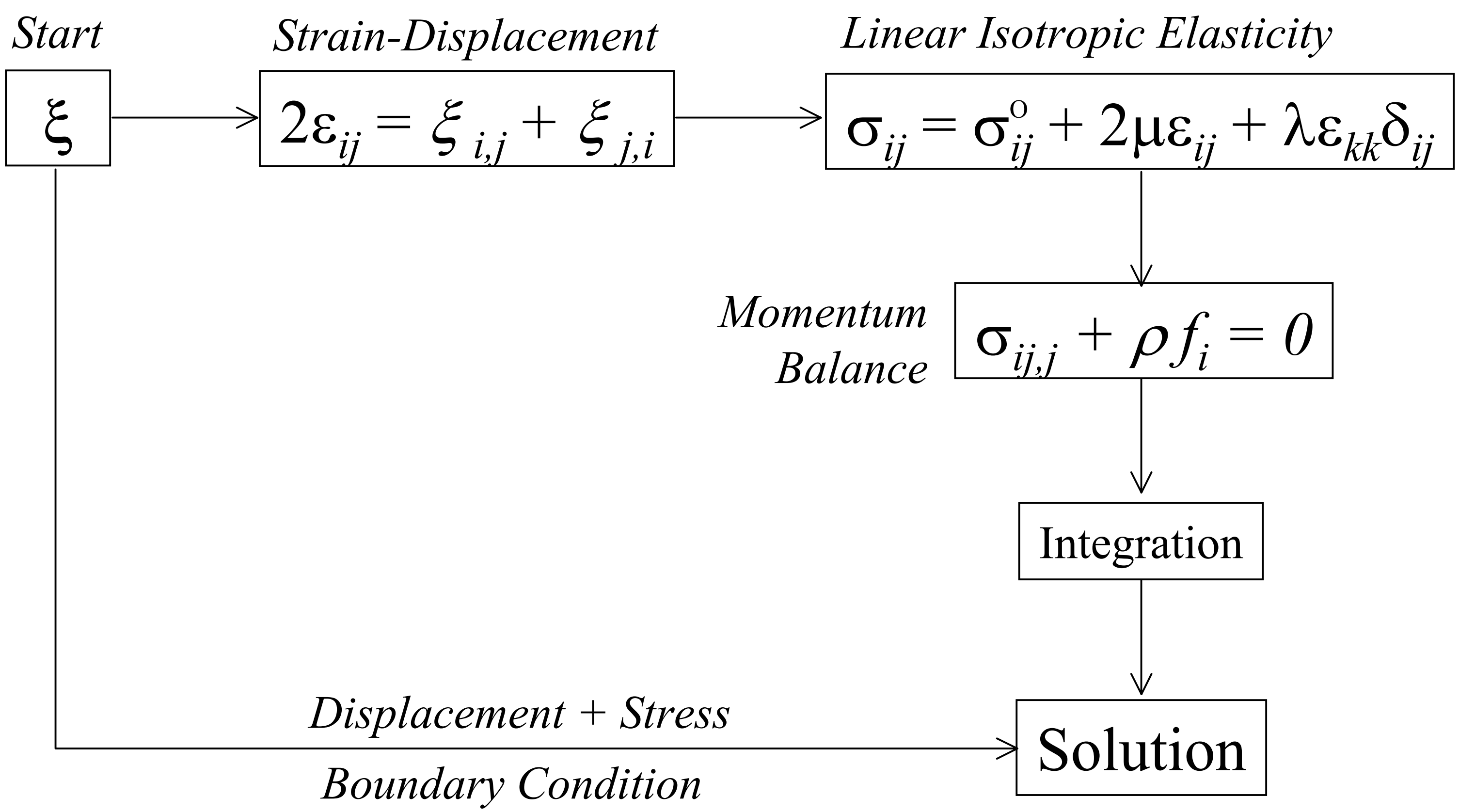


Boussinesq Problem



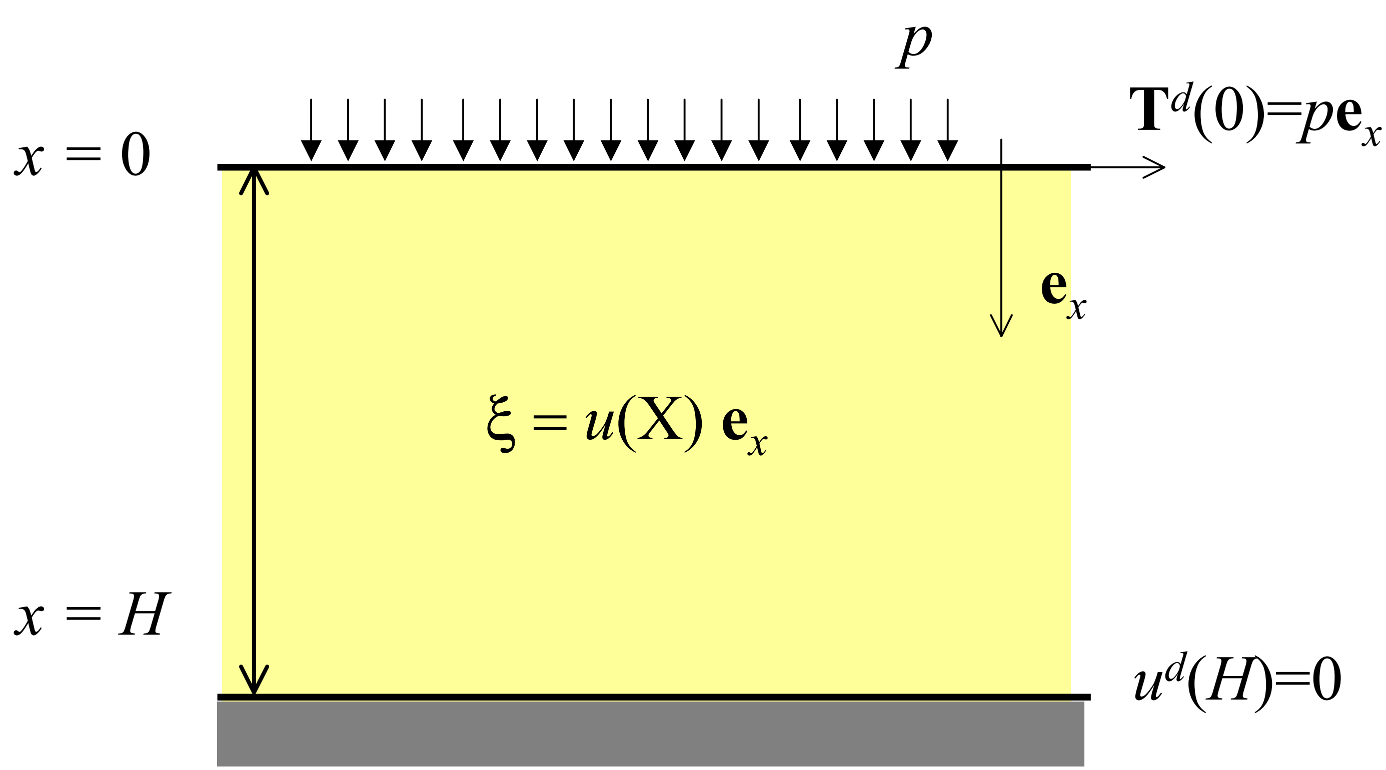
Direct Solving Methods in Elasticity

Displacement Method



Exercise: Soil layer under uniform surface pressure

Application of Displacement Method



Training Set: Cylinder Tube … Deep Tunnel

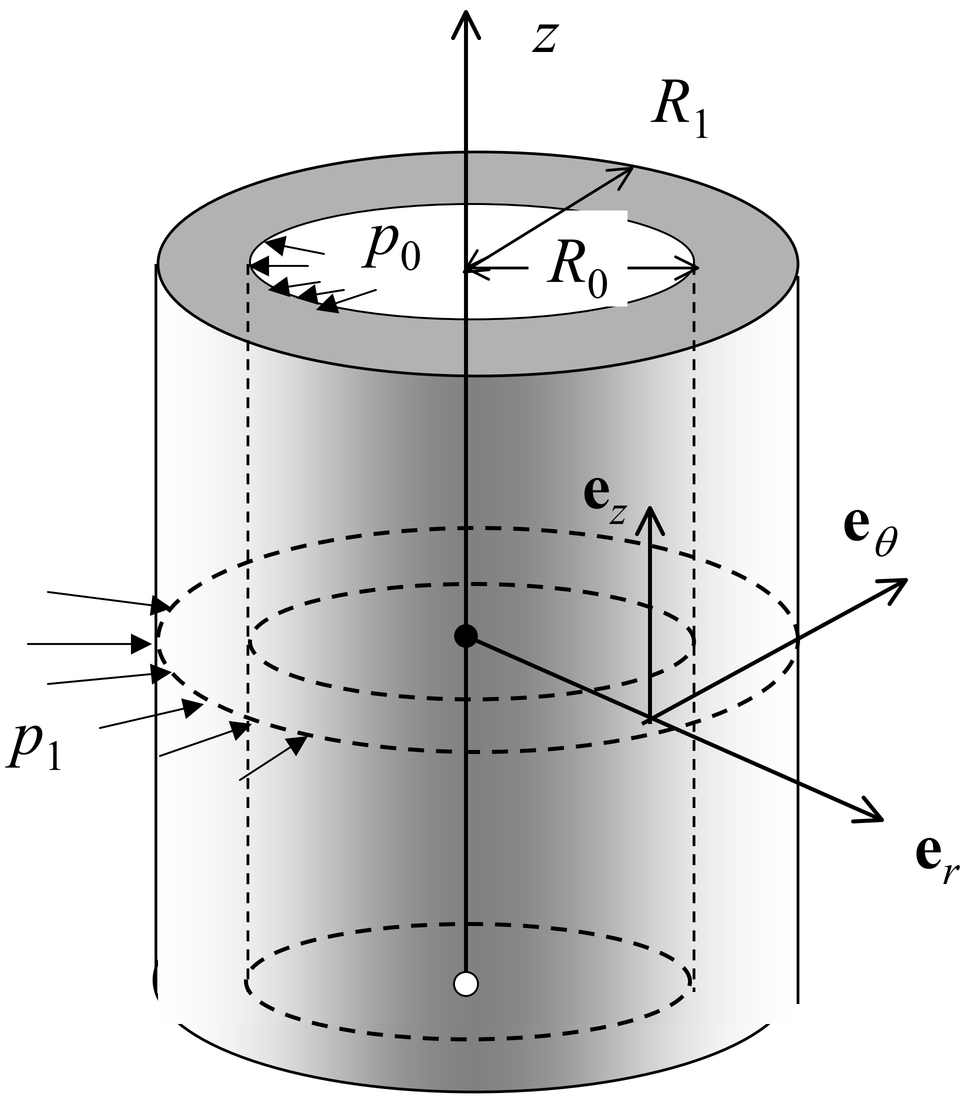
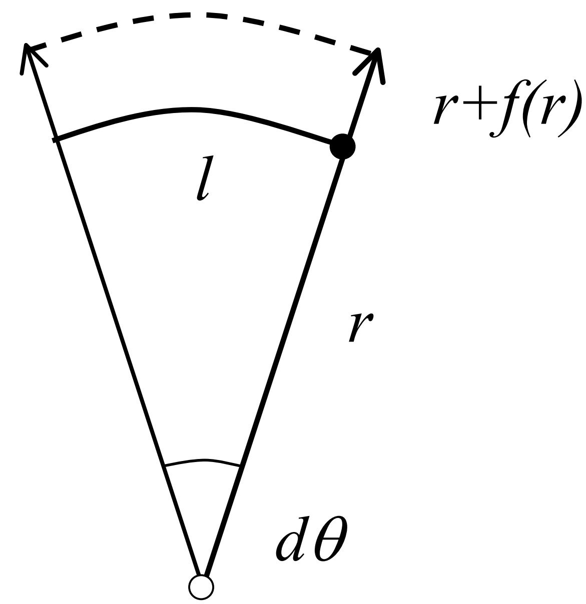


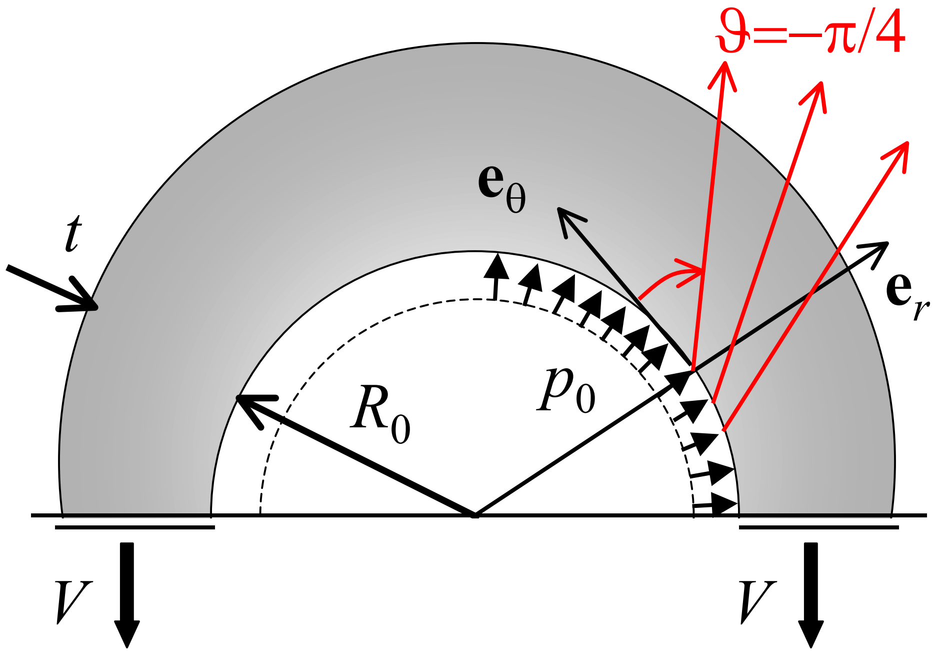
Illustration of cylinder strain components

$l' = l(1+\varepsilon_{\theta\theta})$

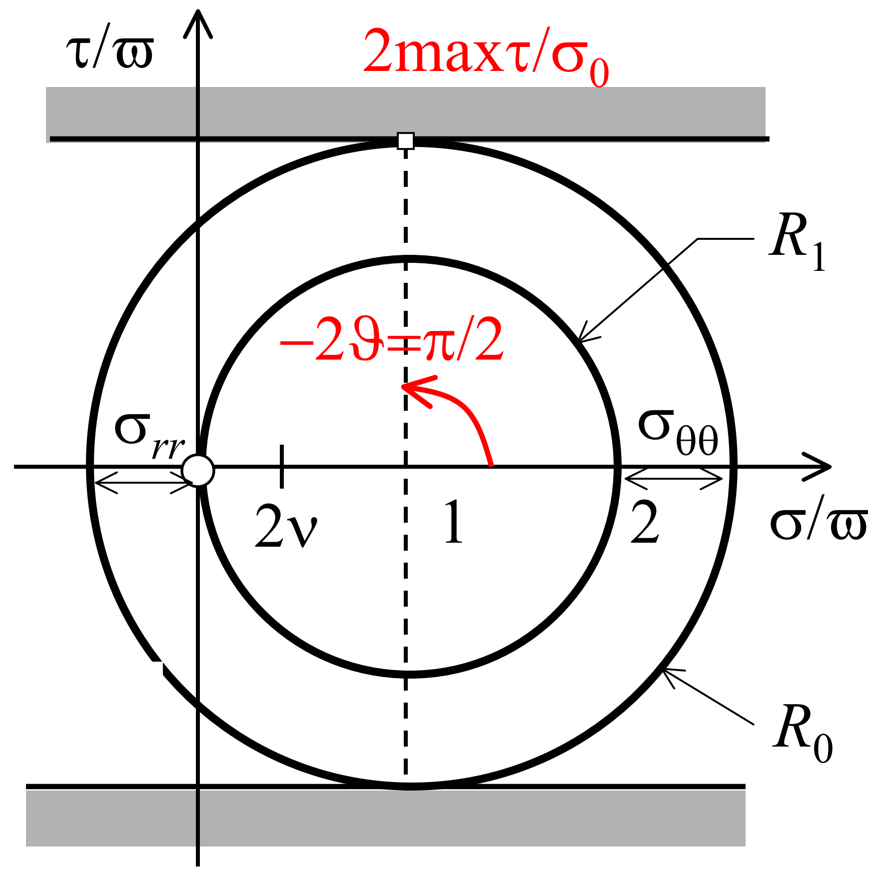


Vessel Formula Revisited

(Elasticity)

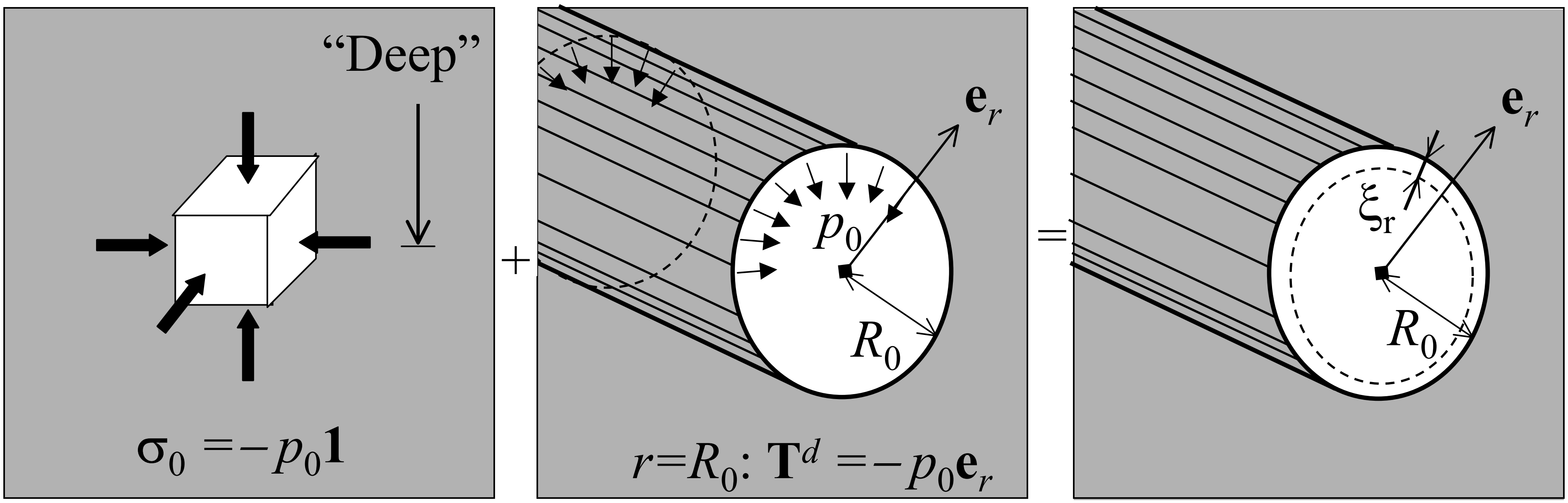


Maximum Shear in Thick Cylinder Tube



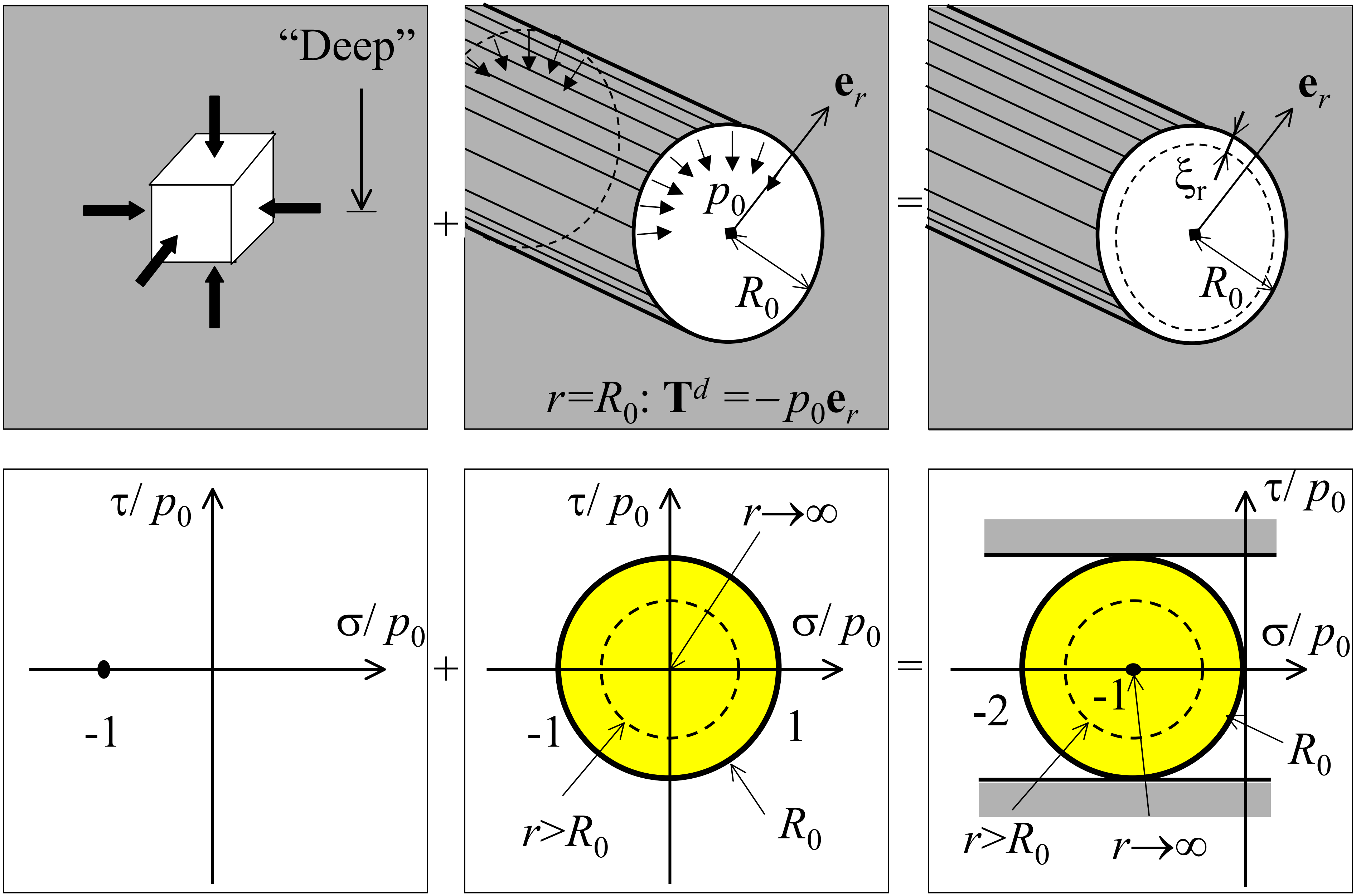
Mohr Representation

Theorem of Superposition
Applied to Deep Tunneling in Elastic Soil/Rock



Natural Isotropic Prestress	+	Excavation = Removal of 'Support'-Stress	=	**Linear Elastic Solution**

Thick tube solution for outer radius $R_1 \to \infty$


"Deep"
+
e_r
p_0
R_0
r=R_0: T^d = −p_0 e_r
=
e_r
ξ_r
R_0
τ/ p_0
σ/ p_0
-1
+
τ/ p_0
r→∞
σ/ p_0
-1
1
r>R_0
R_0
=
τ/ p_0
σ/ p_0
-2
-1
R_0
r>R_0
r→∞