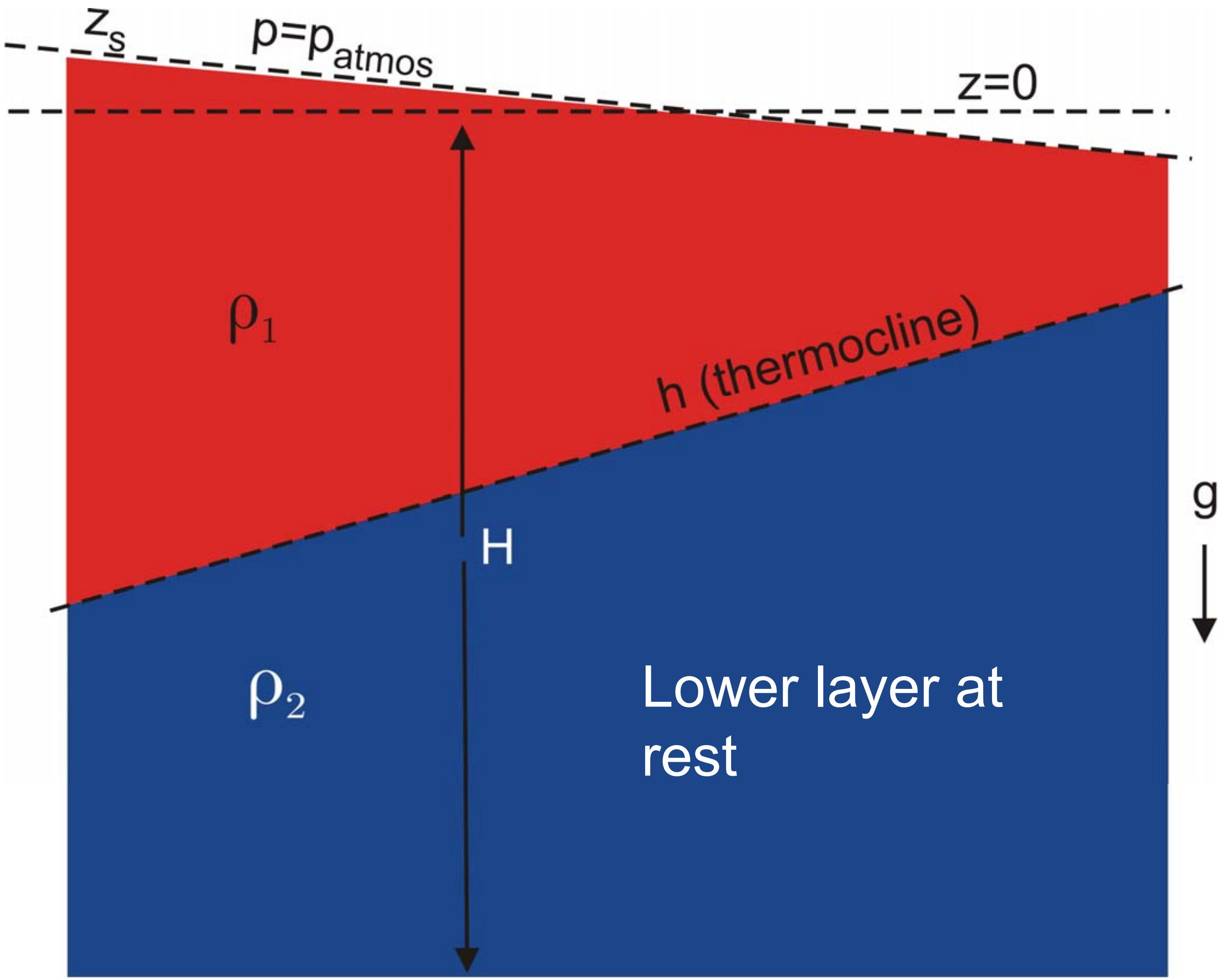
El Niño/Southern Oscillation (ENSO)

Main Issues (contributed by E. Tziperman):

- What is the mechanism of the El Nino cycle?
- Why is the mean period quite robustly 4 years?
- Is ENSO self-sustained or is it damped and requires external forcing by weather noise for example in order to be excited?
- Why are ENSO events irregular: is it due to chaos? noise?
- Why do ENSO events tend to peak toward the end of the calendar year (phase locking to the seasonal cycle)?

ENSO Theory



$$p_H = constant = p_{atmos} + \rho_1 g\left(h + z_s\right) + \rho_2 g\left(H - h\right)$$

$$\rightarrow h\left(\rho_1 - \rho_2\right) + z_s \rho_1 = constant$$

Hydrostatic: At fixed depth within layer 1:

$$p = p_{atmos} + \rho_1 g\left(z + z_s\right)$$

$$\rightarrow \frac{1}{\rho_1}\nabla p = g\nabla z_s = g\frac{\rho_2 - \rho_1}{\rho_1}\nabla h \equiv g^*\nabla h$$

$$\frac{d\mathbf{V}}{dt} + \beta y\hat{k} \times \mathbf{V} + g^*\nabla h = \frac{\boldsymbol{\tau}_s}{\rho_1 h} - \varepsilon_m \mathbf{V}$$

Mass continuity:

$$\int_{-h}^{z_s}\left(\nabla\bullet\mathbf{V}+\frac{\partial w}{\partial z}\right)dz=0$$

$$\rightarrow\frac{dh}{dt}+h\nabla\bullet\mathbf{V}\cong 0$$

Linear shallow water system:

$$\frac{\partial u}{\partial t}-\beta yv+g*\frac{\partial h}{\partial x}=\frac{\tau_x}{\rho H}-\varepsilon_m u,$$

$$\frac{\partial v}{\partial t}+\beta yu+g*\frac{\partial h}{\partial y}=\frac{\tau_y}{\rho H}-\varepsilon_m v,$$

$$\frac{\partial h}{\partial t}+H\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=-\varepsilon_h h$$ ← “relaxation term”

Steady, undamped flow at equator:

$$g^* \frac{\partial h}{\partial x} = \frac{\tau_x}{\rho H}$$

Easterly wind gives decreasing thermocline depth toward east. Shallower thermocline is generally associated with colder surface temperatures

For simplicity, take $\varepsilon_m = \varepsilon_h \equiv \varepsilon$

Curl of momentum equations:

$$\left(\frac{\partial}{\partial t} + \varepsilon\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\beta y h}{H}\right) + \beta v = \hat{k} \bullet \nabla \times \frac{\boldsymbol{\tau}_s}{\rho H}$$

Small variability and damping:

$$\beta v \simeq \hat{k} \bullet \nabla \times \frac{\boldsymbol{\tau}_s}{\rho H}$$ Sverdrup balance

$$\frac{\partial u}{\partial x} \simeq -\frac{\partial v}{\partial y}$$

Steady, undamped flow:

$$g^* \frac{\partial h}{\partial x} = \frac{\tau_x}{\rho H} + \beta y v$$

$$= \frac{\tau_x}{\rho H} - y \frac{1}{\rho H} \frac{\partial \tau_x}{\partial y}$$

Suppose $\frac{\tau_x}{\rho H} = \tau_0 e^{-y^2/2L^2}$

$$\rightarrow g^* \frac{\partial h}{\partial x} = \tau_0 \left(1 + y^2/L^2 \right) e^{-y^2/2L^2}$$

ENSO Theories

- Delayed Oscillator (See E. Tziperman notes)
- Ocean equatorial wave guide (stable or unstable) stochastically forced by atmopshere
- Unstable coupled modes

Rudiments of a local ENSO theory

Consider only (undamped) Kelvin mode in ocean:

$$\frac{\partial u_o}{\partial t} + g^* \frac{\partial h}{\partial x} = \frac{\tau_x}{\rho H},$$

$$\frac{\partial h}{\partial t} + H \frac{\partial u_o}{\partial x} = 0$$

Steady, undamped atmosphere with WISHE, forced by ocean temperature anomalies:

$$\left(T_s - \overline{T}\right)\frac{\partial s^*}{\partial x} = -\beta y v,$$

$$\left(T_s - \overline{T}\right)\frac{\partial s^*}{\partial y} = \beta y u,$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \alpha u = -\gamma s_0.$$

$$\alpha \equiv \frac{\varepsilon_p}{1-\varepsilon_p}\frac{C_k \overline{U}}{h\,|\mathbf{V}|}\frac{\overline{s_0 - s_b}}{\overline{s_b - s_m}}, \quad \gamma \equiv \frac{\varepsilon_p}{1-\varepsilon_p}\frac{C_k\,|\mathbf{V}|}{H\left(\overline{s_b - s_m}\right)}$$

Eliminate *s** and *v* in favor of *u*:

$$\frac{\partial u}{\partial x} + 2\alpha u + \alpha y \frac{\partial u}{\partial y} = -2\gamma s_0 - \gamma y \frac{\partial s_0}{\partial y}$$

Note that on equator:

$$\frac{\partial u}{\partial x} + 2\alpha u = -2\gamma s_0$$

Surface stress and ocean temperature:

$$\tau_x \cong \rho_a C_D \overline{|\mathbf{V}|} u, \qquad s_0 \cong \mu h$$

Ocean:

$$\frac{\partial u_o}{\partial t} + g^* \frac{\partial h}{\partial x} = \frac{\rho_a C_D \overline{|\mathbf{V}|}}{\rho H} u,$$

$$\frac{\partial h}{\partial t} + H \frac{\partial u_o}{\partial x} = 0$$

Atmosphere:

$$\frac{\partial u}{\partial x} + 2\alpha u = -2\gamma\mu h$$

Eliminate variables in favor of *h*:

$$\left(\frac{\partial}{\partial x}+2\alpha\right)\left(\frac{\partial^2 h}{\partial t^2}-g^*H\frac{\partial^2 h}{\partial x^2}\right)-2\gamma H\mu\frac{\rho_a C_D\overline{|\mathbf{V}|}}{\rho H}\frac{\partial h}{\partial x}=0$$

Scalings:

$$c\equiv\sqrt{g^*H}$$

$$\alpha'\equiv a\alpha$$

$$\chi\equiv 2\gamma H\mu\frac{\rho_a C_D\overline{|\mathbf{V}|}}{\rho H a^2 c^2}$$

$$x\rightarrow ax$$

$$t\rightarrow {}^{a}\!/_{c}\,t$$

$$\left(\frac{\partial}{\partial x}+\alpha'\right)\left(\frac{\partial^2 h}{\partial t^2}-\frac{\partial^2 h}{\partial x^2}\right)-\chi\frac{\partial h}{\partial x}=0$$

$$h=h_0 e^{ikx-i\omega t}$$

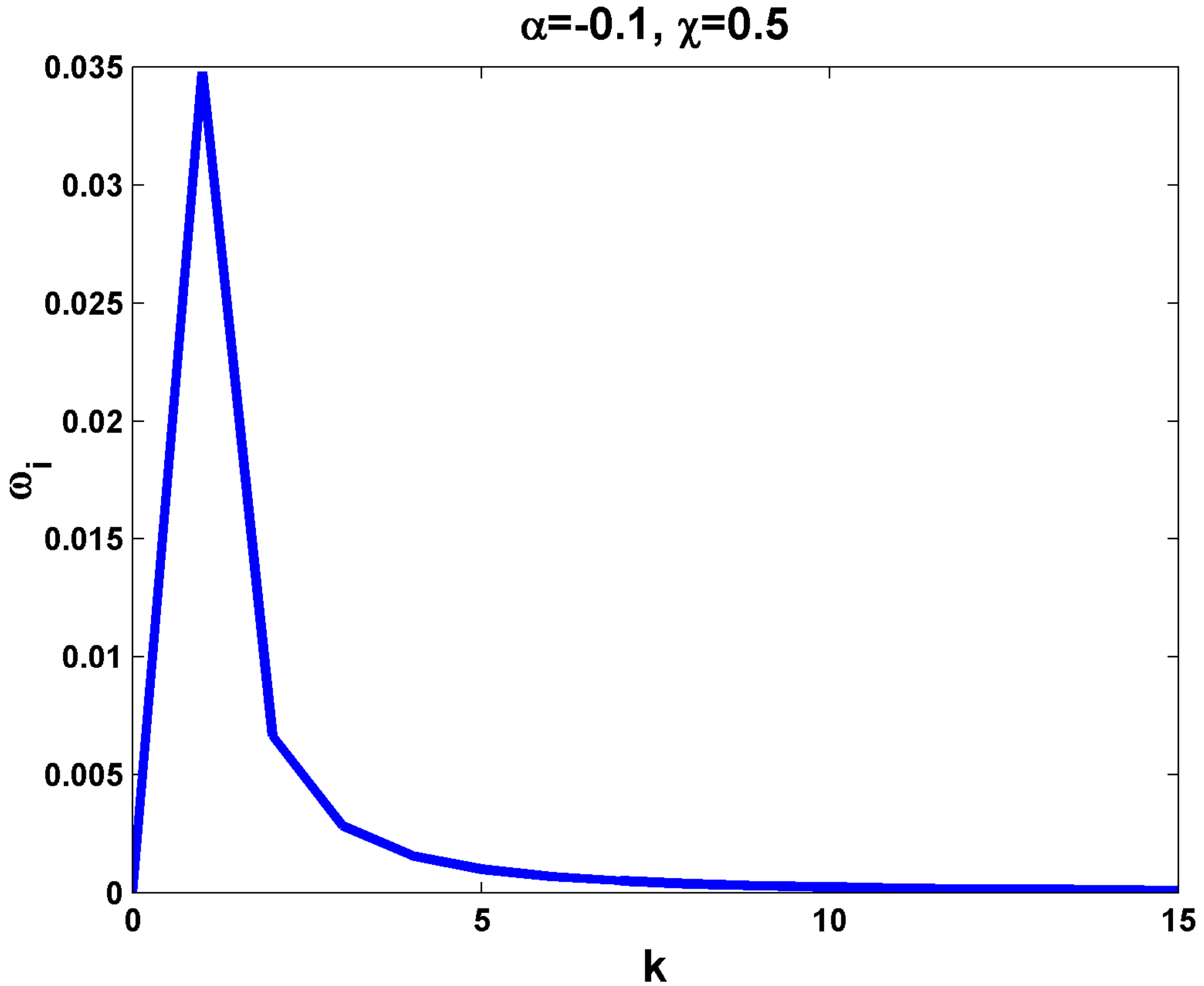
$$\omega^2=k^2-\frac{\chi}{1-i\,\alpha'/k}$$

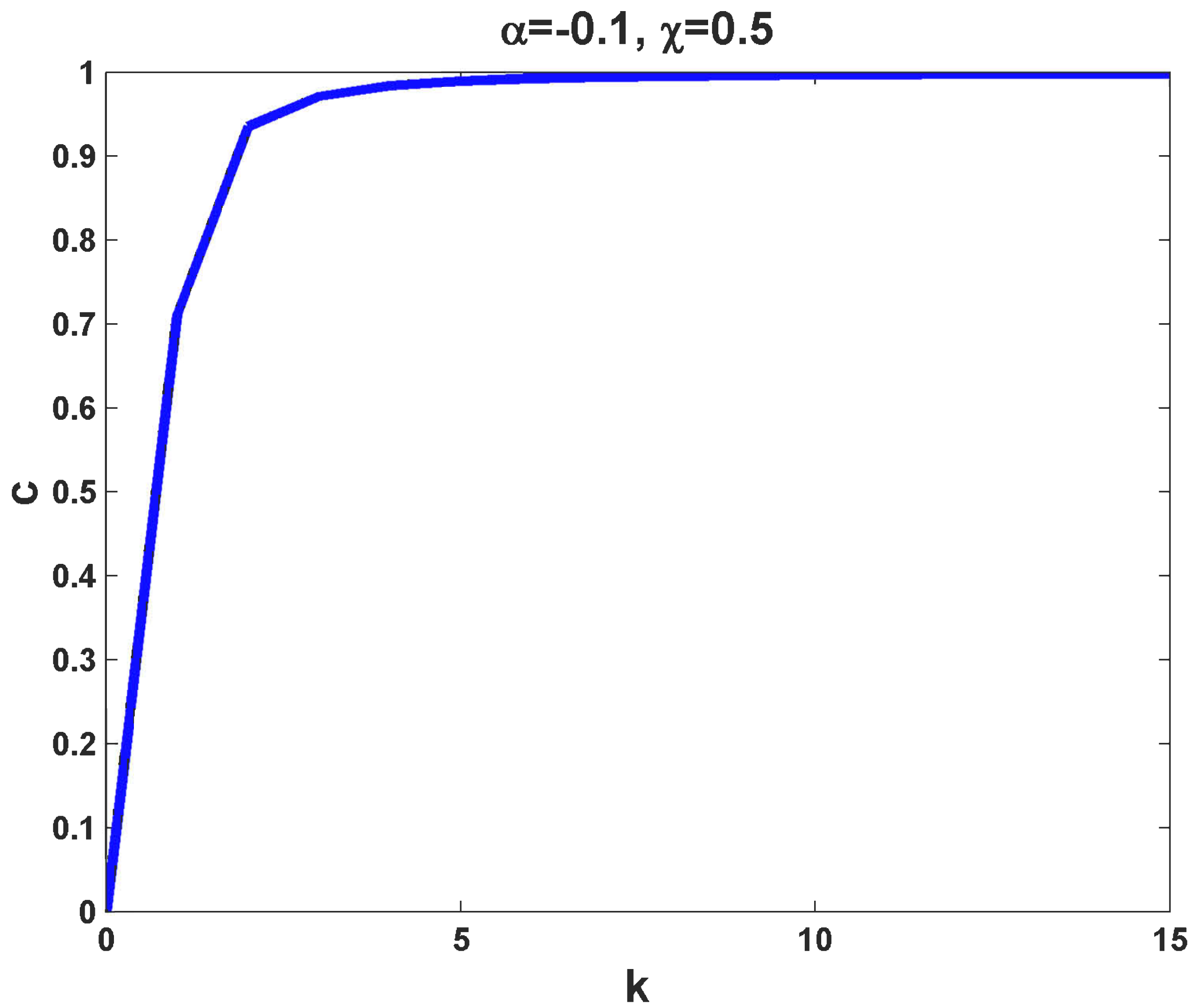
No WISHE ($\alpha'=0$):

$$\omega^2 = k^2 - \chi$$

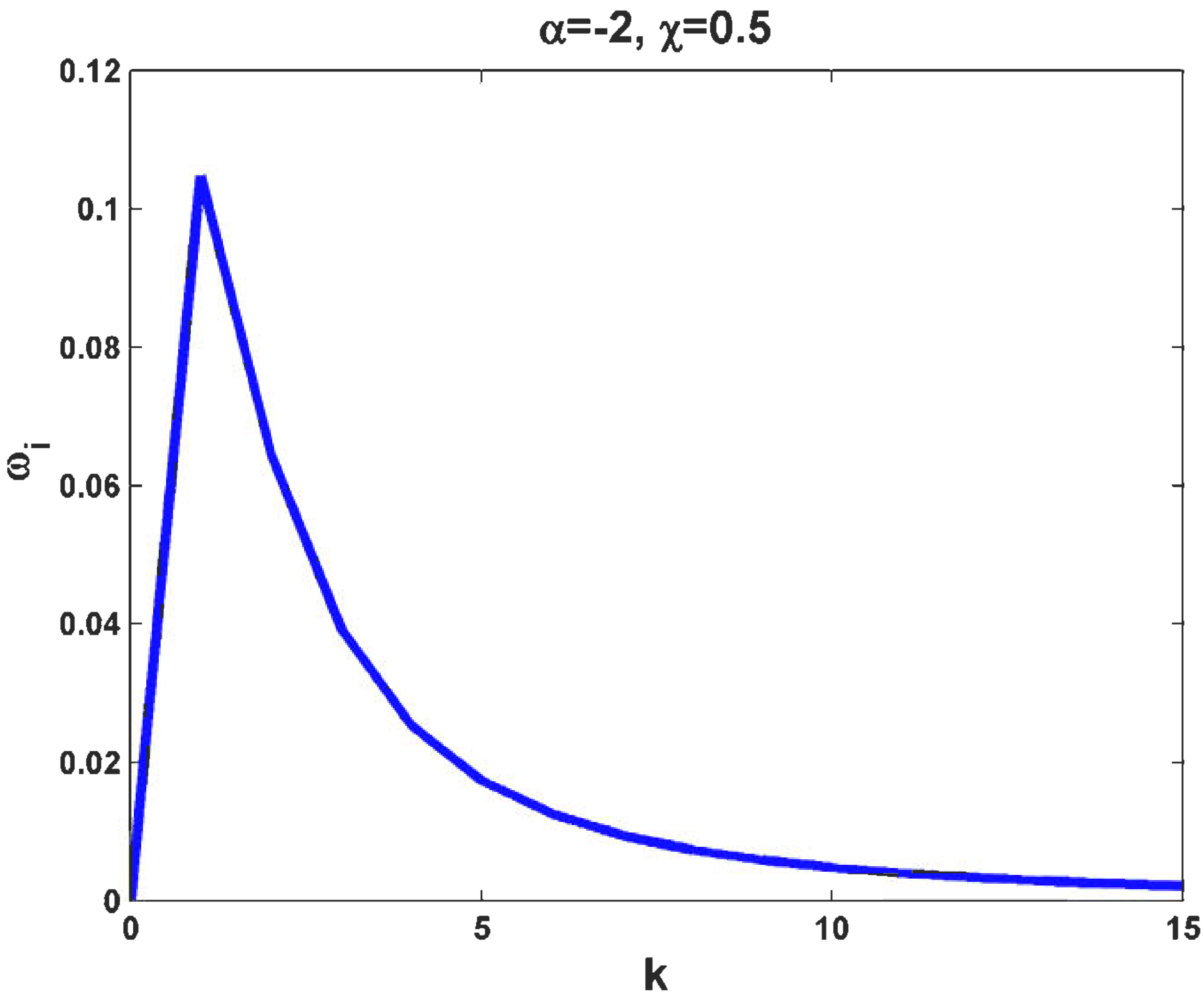
Modes are either neutral and propagating (but slowed down by interaction with atmosphere), or stationary and amplifying/decaying.

With WISHE effect (unstable, eastward-propagating modes only when α’ < 0):

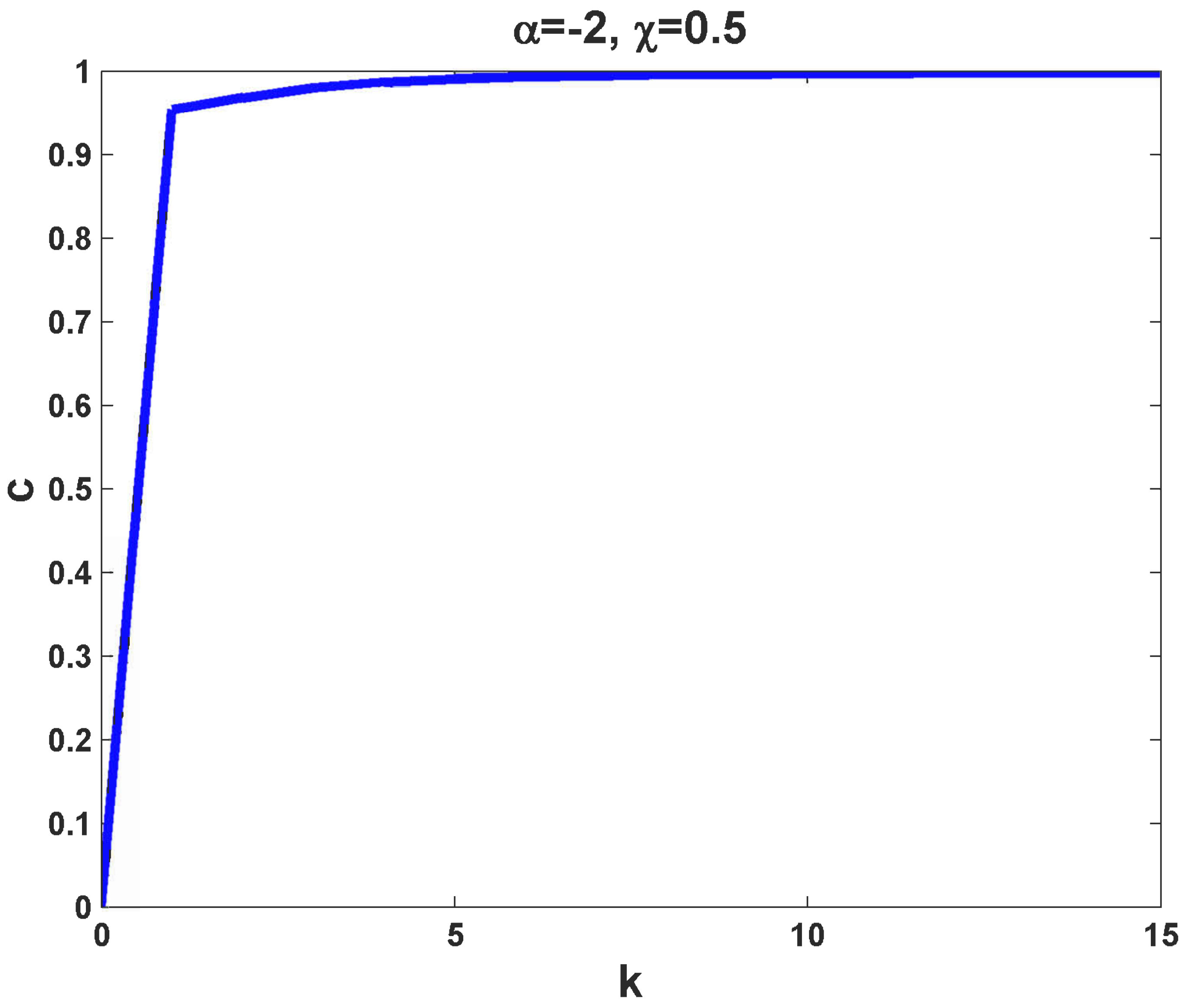




α=-0.1, χ=0.5
c
k
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15



α=-2, χ=0.5
0.12
0.1
0.08
0.06
0.04
0.02
0
ω_i
0
5
10
15
k


α=-2, χ=0.5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
c
0
5
10
15
k

These are only approximate solutions:

- Coupling produces mismatch of y structure...no pure Kelvin modes in ocean
- Advection of temperature by ocean currents ignored
- Mean slope of thermocline ignored
- No damping terms
- No influence of surface flux variations on ocean temperature