# 14.27 Problem Set 4

## Due 11/12

## **Question 1: Online Search Advertising**

#### a)

Find a search term where the sponsored search results (the ads) are quite different on Google and Bing. The differences could be in ad type, quality, quantity, etc. ("Hatch Shell" was the example we saw in class, but choose something different.) What do you think might account for the difference in ads? If it's differences in the quality weighting schemes, speculate about what aspect of quality weighting we discussed in class might be responsible.

#### b)

Find a search term on either Google or Bing which delivers quite different sponsored search results at different times of day. Again, what do you think might account for the difference in ads?

## c)

A large fraction of the sponsored search links that Yahoo! used to serve were for pornography sites, even when search terms had nothing to do with pornography. (Their quality-weighting of ad bids was either very crude or nonexistent at the time.) Why would pornography sites want to do that? What change in policy could Yahoo! have put in place to limit this problem without changing their quality-weighting scheme?

## Question 2: Amazon

### a)

Document the evolution of Amazon's physical presence over time, from when it started operations until today. (The company's Annual Reports would be a great place to start looking.) Describe their location strategy as it related to the laws governing sales tax collection in the United States.

### b)

If all Amazon sales become subject to collection of sales tax (by Amazon for the ship-to state) in the near future, what does the evidence from the economics literature suggest about the futureshape of

Amazon's warehouse and fulfillment network? (Think, in particular, about consumers' tax sensitivity and shipping time sensitivity.)

#### c)

If all Amazon sales become subject to collection of sales tax in the near future (due to political pressure and bilateral negotiations with states), but the same does not happen for small internet retailers, what does the economics literature suggest about the future overall configuration of e- retail? (Here, think also about home state preferences.)

## Question 3: Applying the Revenue Equivalence Theorem

In class we discussed the Revenue Equivalence Theorem, which showed that under a set of standard assumptions many different auction mechanisms raise precisely the same expected revenue for the seller. An additional consequence of the Revenue Equivalence Theorem that we did not discuss is that it also guarantees that in equilibrium, each of these auction mechanisms give the same expected surplus to each bidder, conditional on their valuation v. This is also known as "Payoff Equivalence." In particular, for a bidder with valuation v, the expected utility of this bidder in equilibrium is:

$$EU(v) = \int_{\underline{v}}^{\overline{v}} F(t)^{(n-1)} dt \tag{1}$$

The purpose of this question is to demonstrate how this can be very useful as a shortcut to determining the equilibrium bidding strategies of auction mechanisms that are more complex than our benchmark case, the second price auction, which of course is simply  $b^*(v_i) = v_i$ .

## a)

Write down the equation for the bidder's utility as a function of v, b(v), and F(v) for the first price auction.

#### b)

Use this "Payoff Equivalence" result explained above (Equation 1) combined with your equation in (a) to derive the general expression for the equilibrium bidding function for a bidder in a first price auction. (This would have been very difficult for the first price auction without this trick.)

#### c)

Now let F() be specified as U[0,1]. Plug this into part (b) to derive the equation that the Professor gave in class for the first price auction equilibrium bidding strategy.

Now consider a "Sealed Bid All Pay Auction," where each of the N bidders submits their bid, the highest bid wins the object, but every single bidder must pay their submitted bid even if they don't win the object.

## d)

Write down the equation for the bidder's utility as a function of v, b(v), and F(v) for the all pay auction.

## **e**)

Use this "Payoff Equivalence" result explained above (Equation 1) combined with your equation in (d) to derive the general expression for the equilibrium bidding function for a bidder in the all pay auction.

### f)

Now let F() be specified as U[0, 1]. Calculate the expected revenue to the seller in the all pay auction.

## g)

Despite these revenue and payoff equivalence results, why do you think that the all pay auction is almost never used in real life?

Fun note: the all pay auction serves as a very nice model of political lobbying / bribery since interest groups on all sides pay their sunk costs in lobbying fees and yet only one decision can ultimately be made on an issue.

## **Question 4: Second Degree Price Discrimination**

A monopolist is studying different pricing schemes in a market. It knows that there are two types of consumers in the market, 1 and 2, in equal proportion. Consumer type 1 is willing to pay \$20 for a high quality unit and \$10 for a low quality unit, while consumer type 2 is willing to pay \$13 for a high quality unit and \$9 for a low quality unit. Assume that the marginal cost of production is 0.

## a)

If the monopolist offers a single product, what quality level does it choose and what price does it charge?

## b)

If the monopolist implements a second degree price discrimination scheme by offering both quality levels at different prices  $((p_H, q_H), (p_L, q_L))$ , what prices and quality levels does it set?

## **c**)

Compare profits and consumer surplus under both settings. What scheme is socially optimal? Give intuition for your answer.

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