Recitation 9 October 7, 2010

- 1. Let X be an exponential random variable with parameter $\lambda > 0$. Calculate the probability that X belongs to one of the intervals [n, n + 1] with n odd.
- 2. (Example 3.13 of the text book, page 165) **Exponential Random Variable is Memoryless.** The time T until a new light bulb burns out is an exponential random variable with parameter λ . Ariadne turns the light on, leaves the room, and when she returns, t time units later, finds that the bulb is still on, which corresponds to the event $A = \{T > t\}$. Let X be the additional time until the bulb burns out. What is the conditional CDF of X, given the event A?
- 3. Problem 3.23, page 191 in the text. Let the random variables X and Y have a joint PDF which is uniform over the triangle with vertices (0,0), (0,1), and (1,0).
 - (a) Find the joint PDF of X and Y.
 - (b) Find the marginal PDF of Y.
 - (c) Find the conditional PDF of X given Y.
 - (d) Find $\mathbf{E}[X \mid Y = y]$, and use the total expectation theorem to find $\mathbf{E}[X]$ in terms of $\mathbf{E}[Y]$.
 - (e) Use the symmetry of the problem to find the value of $\mathbf{E}[X]$.
- 4. We have a stick of unit length, and we break it into three pieces. We choose randomly and independently two points on the stick using a uniform PDF, and we break the stick at these points. What is the probability that the three pieces we are left with can form a triangle?

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