# 6.641 Electromagnetic Fields, Forces, and Motion Spring 2009

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#### Massachusetts Institute of Technology Department of Electrical Engineering and Computer Science 6.641 Electromagnetic Fields, Forces, and Motion

Problem Set #1	Issued:
Spring Term 2009	Due:
<u>Suggested Reading</u> : Zahn – Section 1.4, 1.5, 2.1-2.4, 3.3, 5.1, 5.2, 5.6	
<u>View Videos</u> – 1.3.1, 1.5.1, 1.4.1, 1.6.1	

#### Problem 1.1

In Demonstration 1.4.1 the magnetic field strength is measured by a Hall effect probe. This probe works by the principle that when charges flow perpendicular to a magnetic field, the transverse displacement due to the Lorentz force can give rise to an electric field.



Figure 5-6 A magnetic field perpendicular to a current flow deflects the charges transversely giving rise to an electric field and the Hall voltage. The polarity of the voltage is the same as the sign of the charge carriers.

(a) A uniform magnetic field  $\overline{B} = B_o \overline{i}_z = \mu_o H_o \overline{i}_z$  is applied to a material carrying a current in the *y* direction. For positive charges, as for holes in a p-type

semiconductor, the charge velocity  $\overline{v} = v_y \overline{l}_y$  is in the positive y direction, while for negative charges as typically occur in metals or in n-type semiconductors, the charge velocity  $v_y$  is negative. In the steady state, the charge velocity  $v_y$  does not vary with time so the net force on the charges must be zero. What is the electric field (magnitude and direction) in terms of  $v_y$  and  $B_o$ ?

(b) What is the Hall voltage,  $V_H = \Phi(x = d) - \Phi(x = 0)$  in terms of  $v_y$ ,  $B_o$  and d?

(c) Can this measurement determine the polarity of the charge carriers assuming that the current *i* is positively *y*-directed and  $B_0$  is positively *z*-directed?

## Problem 1.2

In a spherically symmetric configuration, the region r < b has the non-uniform charge density  $\rho_b r/b$  and is surrounded by a region b < r < a having the uniform charge density  $\rho_a$ . At r = b there is no surface charge density, while at r = a there is perfectly conducting sheet with surface charge density that assures E = 0 for r > a.

- (a) What is the total charge in the regions 0 < r < b and b < r < a?
- (b) Determine E in the two regions r < b and b < r < a.
- (c) What is the surface charge density at r = a?
- (d) What is the total charge in the system for  $0 < r \le a$ .

### Problem 1.3

In polar coordinates, a non-uniform current density  $J_0 r / b \overline{i_z}$  exists over the cross-section of a wire having a radius *b*. The total current in the wire is returned in the -z direction as a uniform surface current at the radius r = a where a > b.

- (a) What is the surface current density at r = a?
- (b) Find the magnetic field in the regions 0 < r < b, b < r < a, and r > a.



- (a) Two point charges are a distance d apart along the y-axis. Find the electric field (magnitude and direction) at any point in the y=0 plane when the charges are:
  - i)  $q_1 = q$ ,  $q_2 = 0$
  - ii) both equal  $q_1 = q_2 = q$

iii) of opposite polarity but equal magnitude  $q_1 = -q_2 = q$ .

This configuration is called an electric dipole with dipole moment  $p_y = qd$ .

(b) For each of the 3 cases in (a) find the force on  $q_1$ .