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High Speed Communication Circuits
Lecture 17
Noise in Voltage Controlled Oscillators

Michael Perrott
Massachusetts Institute of Technology
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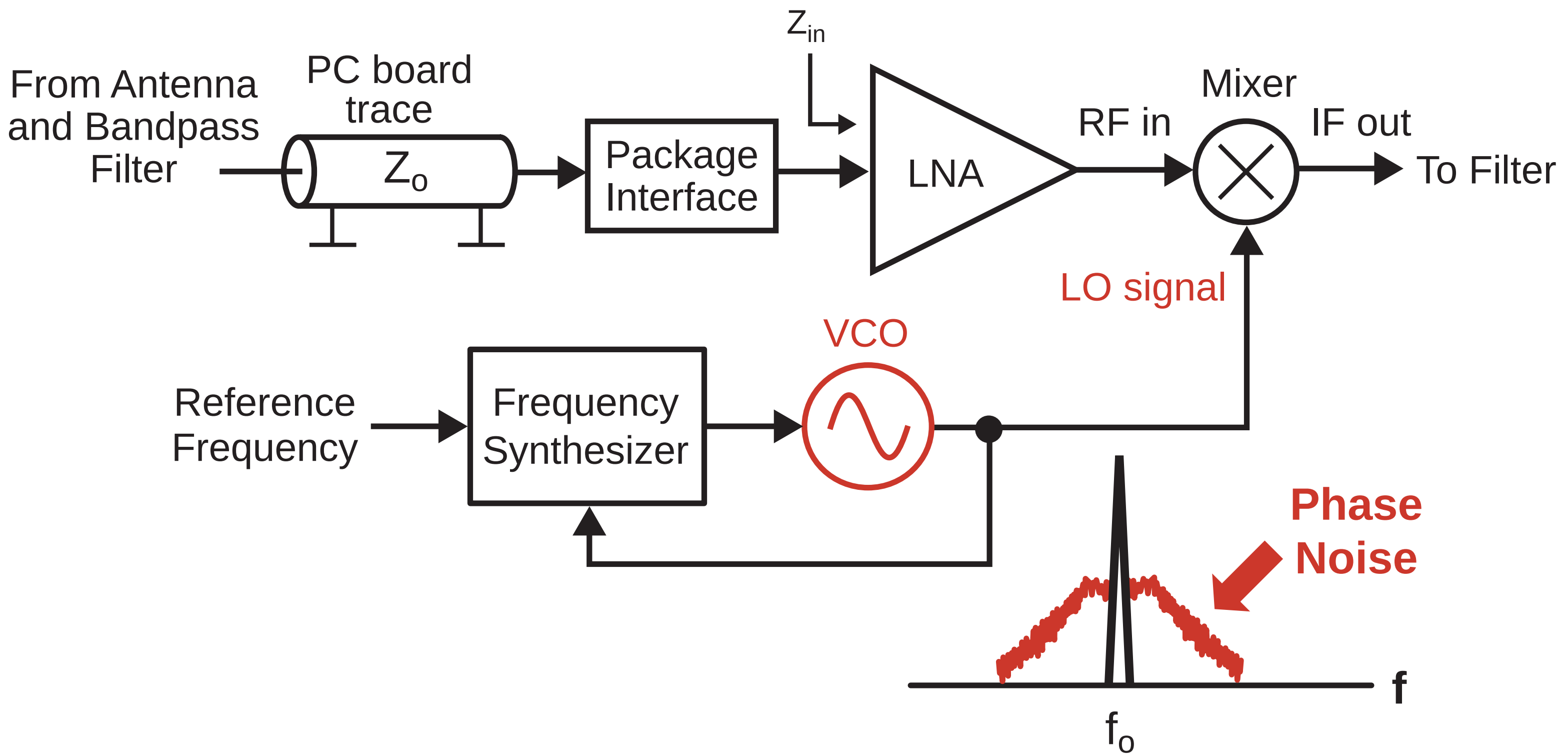
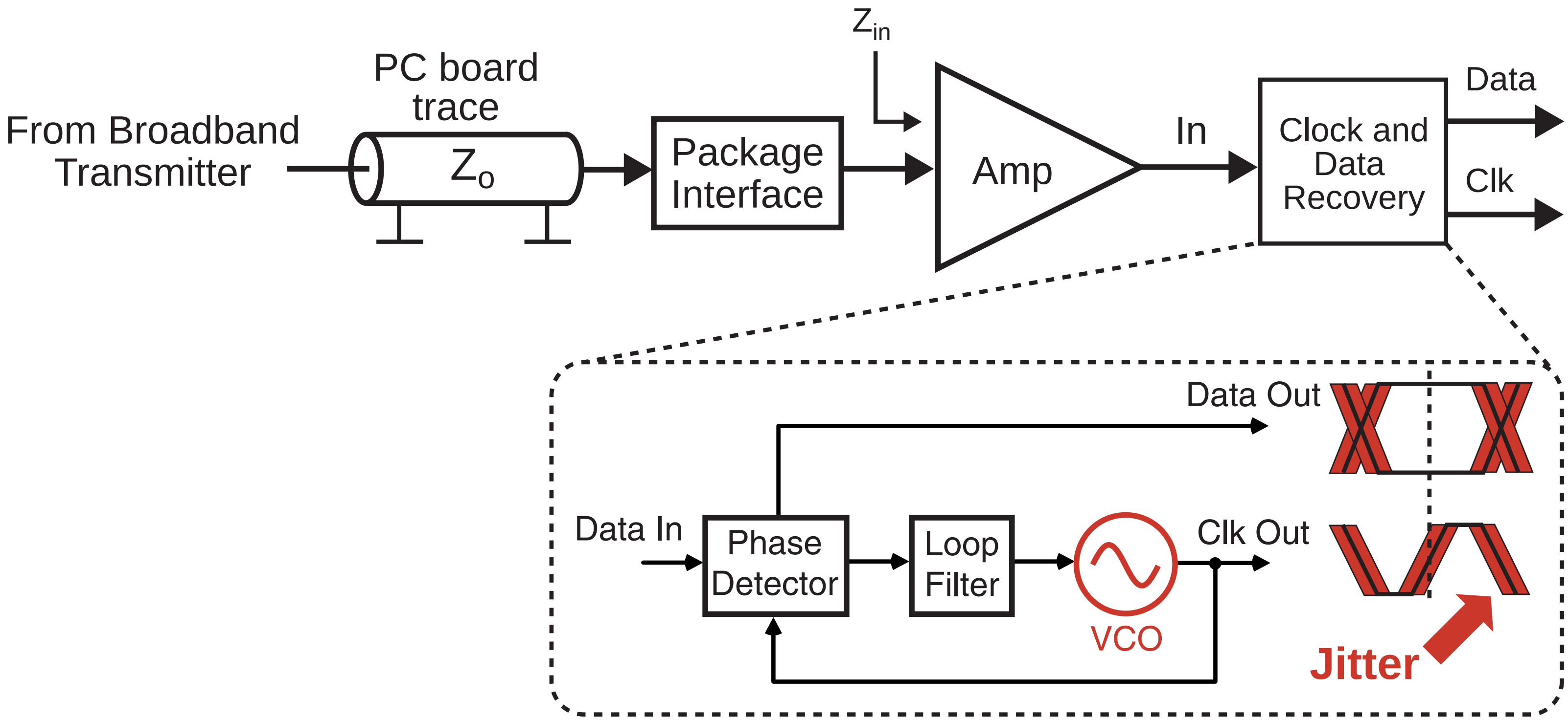
VCO Noise in Wireless Systems



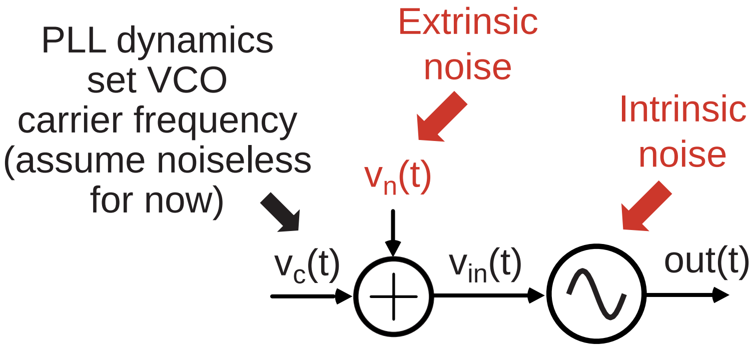
- **VCO noise has a negative impact on system performance**
  - **Receiver – lower sensitivity, poorer blocking performance**
  - **Transmitter – increased spectral emissions (output spectrum must meet a mask requirement)**
- **Noise is characterized in frequency domain**

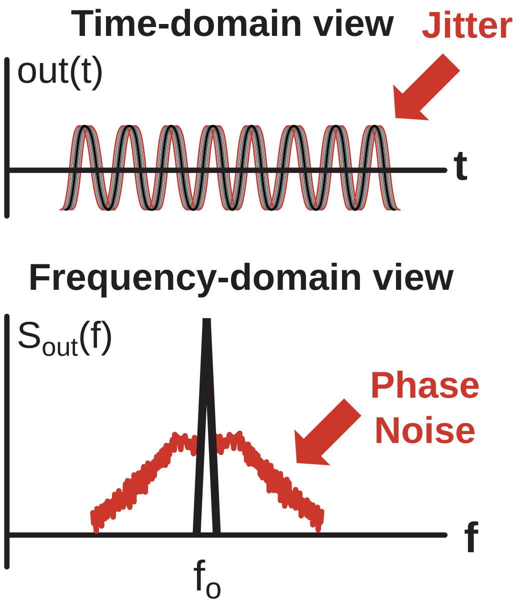
VCO Noise in High Speed Data Links



- **VCO noise also has a negative impact on data links**
  - **Receiver – increases bit error rate (BER)**
  - **Transmitter – increases jitter on data stream (transmitter must have jitter below a specified level)**
- **Noise is characterized in the time domain**

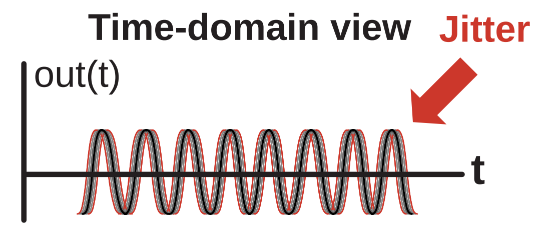
Noise Sources Impacting VCO



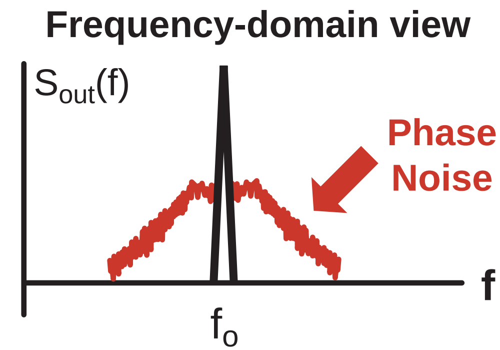


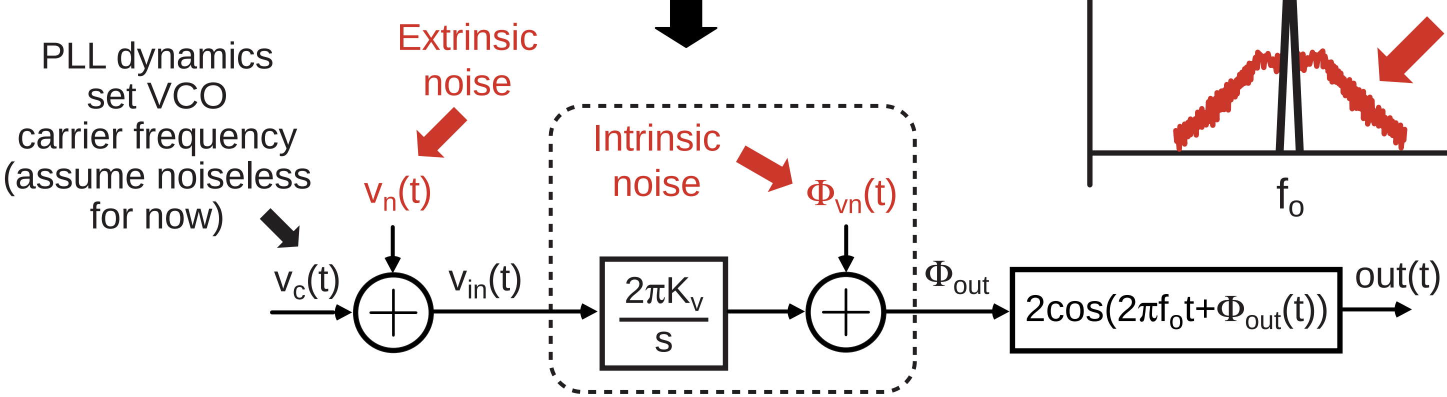
- **Extrinsic noise**
  - **Noise from other circuits (including PLL)**
- **Intrinsic noise**
  - **Noise due to the VCO circuitry**

VCO Model for Noise Analysis



Note: K_v units are Hz/V

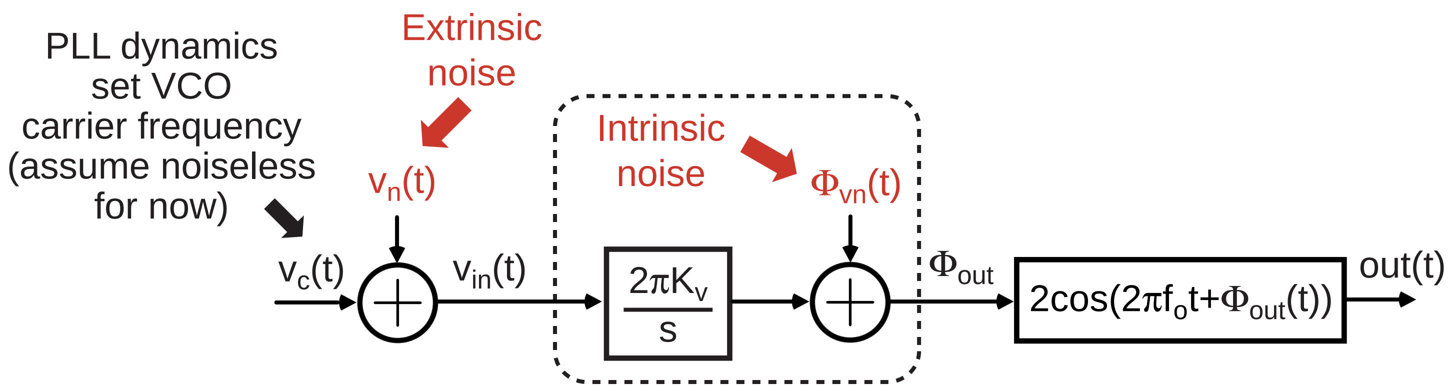




- **We will focus on phase noise (and its associated jitter)**
  - **Model as phase signal in output sine waveform**

$$out(t) = 2\cos(2\pi f_o t + \underline{\Phi_{out}(t)})$$

Simplified Relationship Between Φ_{out} and Output



$$out(t) = 2\cos(2\pi f_o t + \Phi_{out}(t))$$

- **Using a familiar trigonometric identity**

$$out(t) = 2\cos(2\pi f_o t)\cos(\Phi_{out}(t)) - 2\sin(2\pi f_o t)\sin(\Phi_{out}(t))$$

- **Given that the phase noise is small**

$$\cos(\Phi_{out}(t)) \approx 1, \quad \sin(\Phi_{out}(t)) \approx \Phi_{out}(t)$$

$$\Rightarrow \quad out(t) = 2\cos(2\pi f_o t) - 2\sin(2\pi f_o t)\Phi_{out}(t)$$

Calculation of Output Spectral Density

$$out(t) = 2\cos(2\pi f_o t) - 2\sin(2\pi f_o t)\Phi_{out}(t)$$

- **Calculate autocorrelation**

$$R\{out(t)\} = R\{2\cos(2\pi f_o t)\} + R\{2\sin(2\pi f_o t)\} \cdot R\{\Phi_{out}(t)\}$$

- **Take Fourier transform to get spectrum**

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$

  - **Note that * symbol corresponds to convolution**
- **In general, phase spectral density can be placed into one of two categories**
  - **Phase noise – Φ_{out}(t) is non-periodic**
  - **Spurious noise - Φ_{out}(t) is periodic**

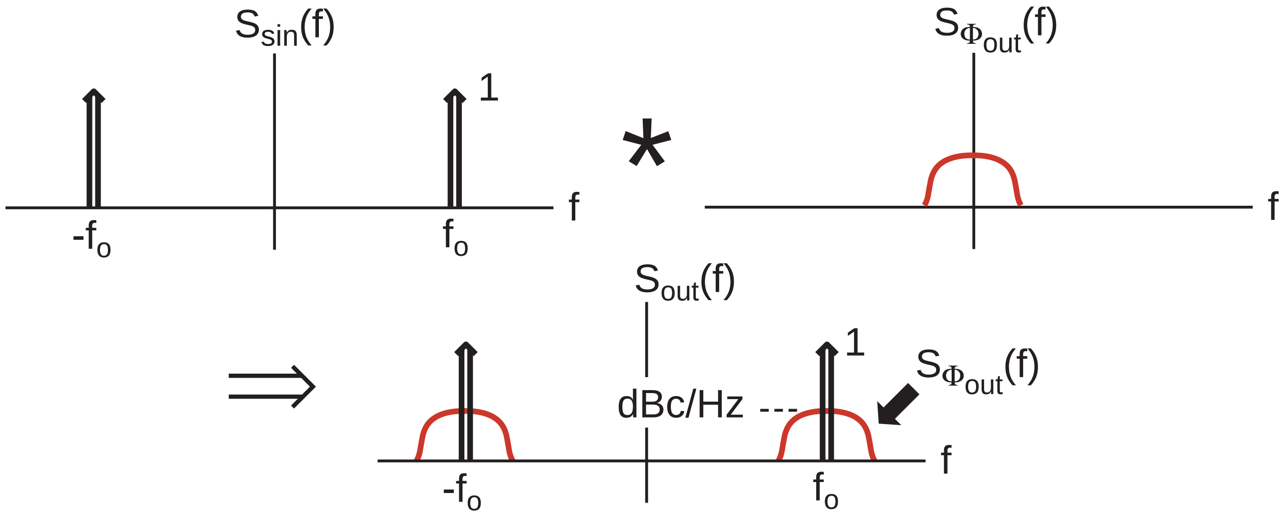
Output Spectrum with Phase Noise

- **Suppose input noise to VCO ($v_n(t)$) is bandlimited, non-periodic noise with spectrum $S_{vn}(f)$**
  - **In practice, derive phase spectrum as**

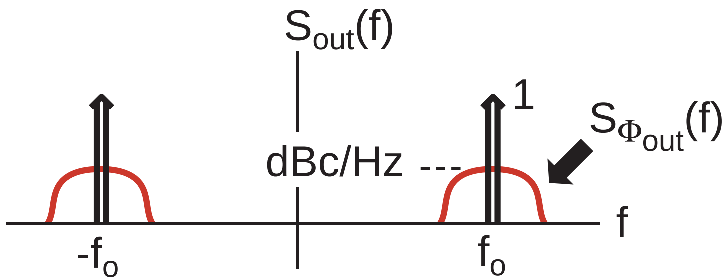
$$S_{\Phi_{out}}(f) = \left(\frac{K_v}{f}\right)^2 S_{v_n}(f)$$

- **Resulting output spectrum**

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$



Measurement of Phase Noise in dBc/Hz



- **Definition of L(f)**

$$L(f) = 10\log\left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}}\right)$$

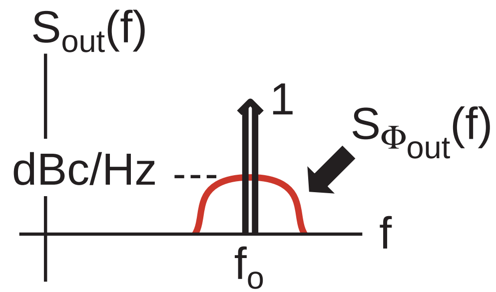
  - **Units are dBc/Hz**

- **For this case**

$$L(f) = 10\log\left(\frac{2S_{\Phi_{out}}(f)}{2}\right) = 10\log(S_{\Phi_{out}}(f))$$

  - **Valid when $\Phi_{out}(t)$ is small in deviation (i.e., when carrier is not modulated, as currently assumed)**

Single-Sided Version



- **Definition of L(f) remains the same**

$$L(f) = 10\log\left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}}\right)$$

  - **Units are dBc/Hz**

- **For this case**

$$L(f) = 10\log\left(\frac{S_{\Phi_{out}}(f)}{1}\right) = 10\log(S_{\Phi_{out}}(f))$$

  - **So, we can work with either one-sided or two-sided spectral densities since L(f) is set by *ratio* of noise density to carrier power**

Output Spectrum with Spurious Noise

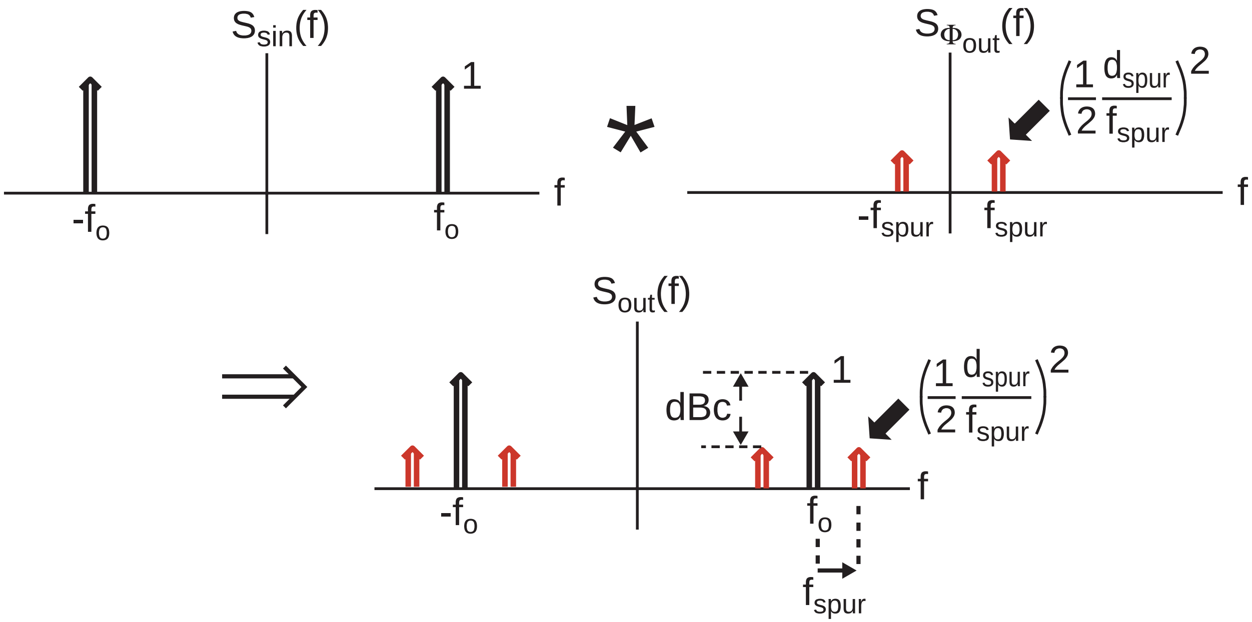
- **Suppose input noise to VCO is**

$$v_n(t) = \frac{d_{spur}}{K_v}\cos(2\pi f_{spur} t)$$

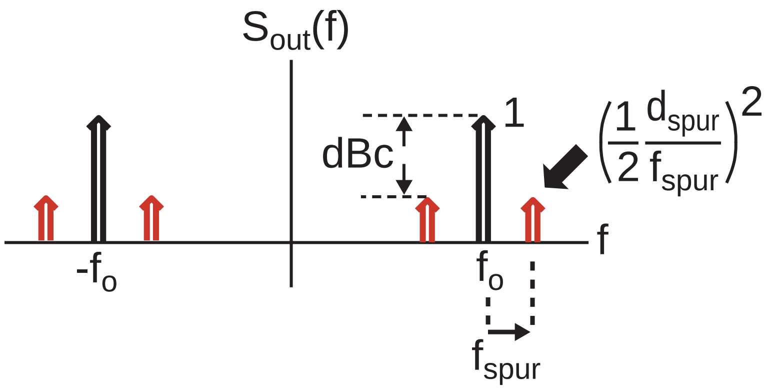
$$\Rightarrow \quad \Phi_{out}(t) = 2\pi K_v \int v_n(t)dt = \frac{d_{spur}}{f_{spur}}\sin(2\pi f_{spur} t)$$

- **Resulting output spectrum**

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$



Measurement of Spurious Noise in dBc



- **Definition of dBc**

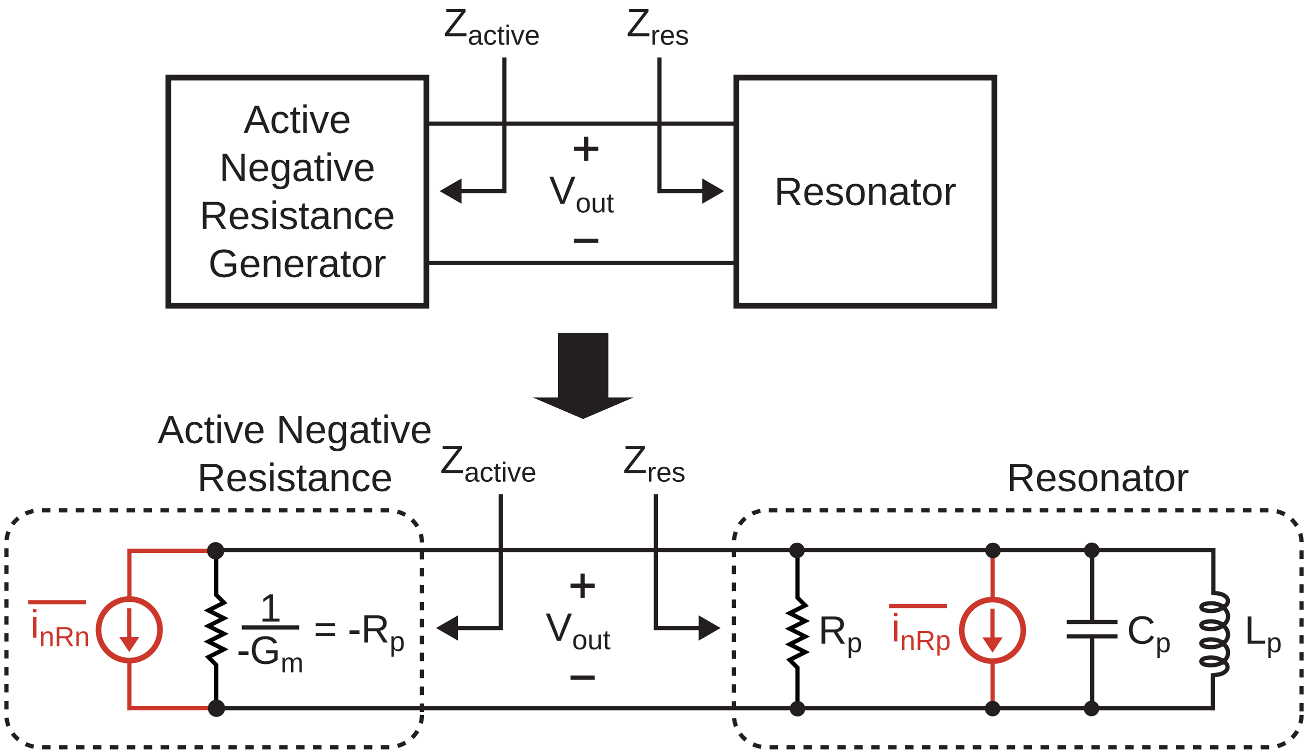
$$10\log\left(\frac{\text{Power of tone}}{\text{Power of carrier}}\right)$$

  - **We are assuming double sided spectra, so integrate over positive and negative frequencies to get power**
    - Either single or double-sided spectra can be used in practice

- **For this case**

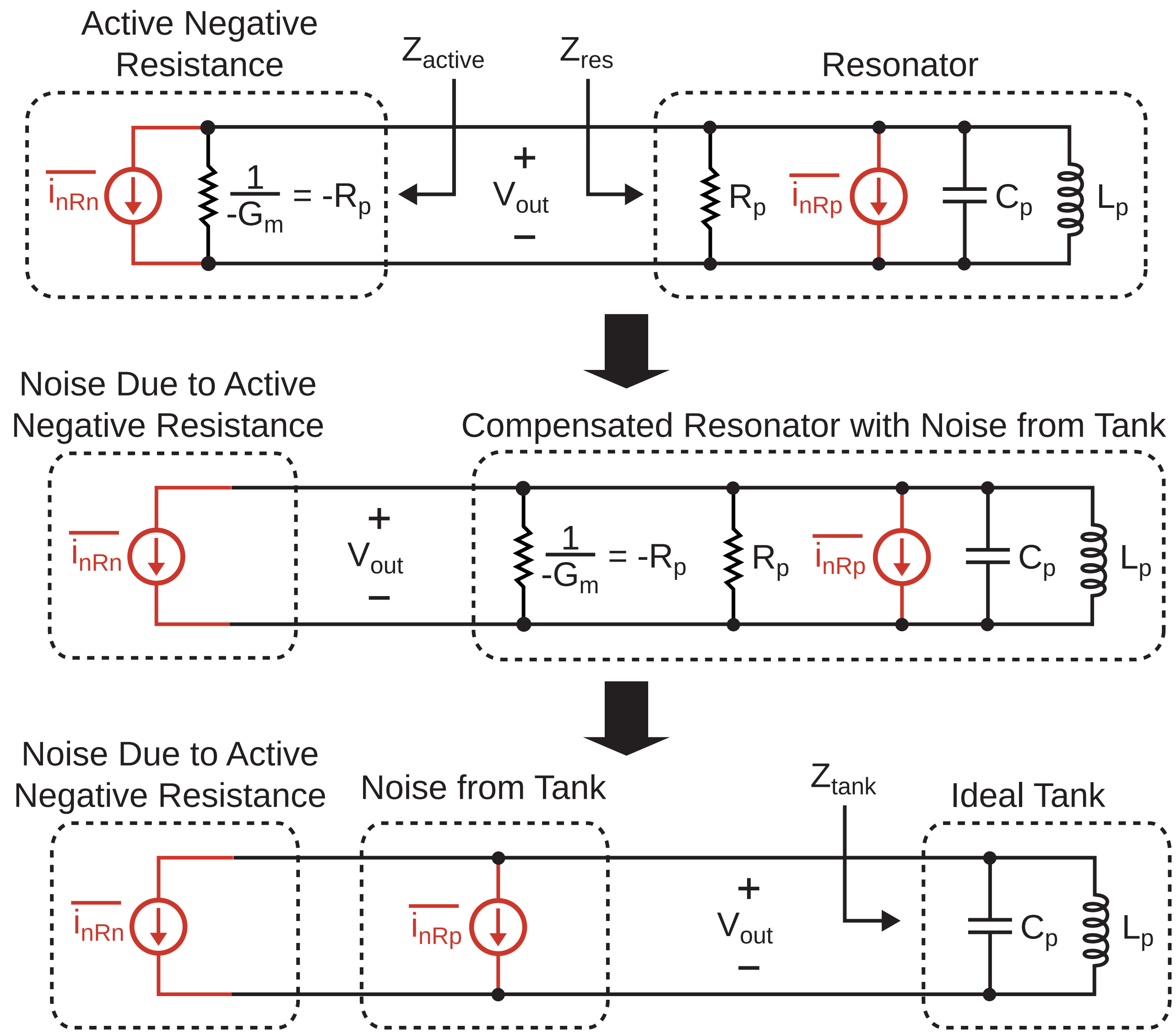
$$10\log\left(\frac{2(\frac{d_{spur}}{2f_{spur}})^2}{2}\right) = 20\log\left(\frac{d_{spur}}{2f_{spur}}\right)\ dBc$$

Calculation of Intrinsic Phase Noise in Oscillators

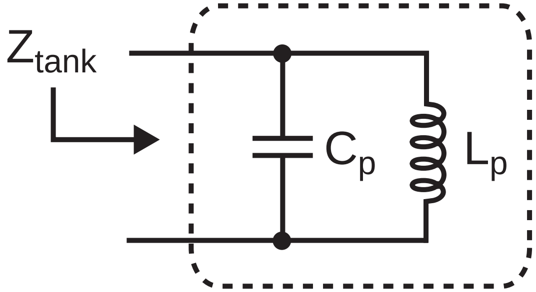


- **Noise sources in oscillators are put in two categories**
  - **Noise due to tank loss**
  - **Noise due to active negative resistance**
- **We want to determine how these noise sources influence the phase noise of the oscillator**

Equivalent Model for Noise Calculations



Calculate Impedance Across Ideal LC Tank Circuit



$$Z_{tank}(w) = \frac{1}{jwC_p} || jwL_p = \frac{jwL_p}{1 - w^2 L_p C_p}$$

- **Calculate input impedance about resonance**

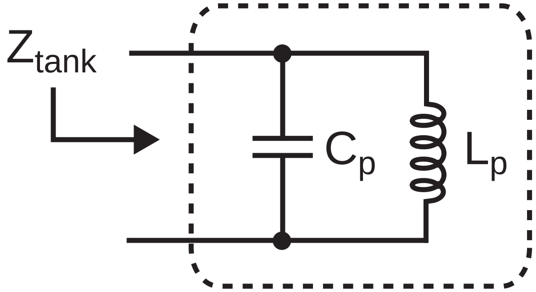
$$\text{Consider } w = w_o + \Delta w, \text{ where } w_o = \frac{1}{\sqrt{L_p C_p}}$$

$$Z_{tank}(\Delta w) = \frac{j(w_o + \Delta w)L_p}{1 - (w_o + \Delta w)^2 L_p C_p}$$

$$= \frac{j(w_o + \Delta w)L_p}{\underbrace{1 - w_o^2 L_p C_p}_{=0} - 2\Delta w(w_o L_p C_p) - \underbrace{\Delta w^2 L_p C_p}_{\text{negligible}}} \approx \frac{j(w_o + \Delta w)L_p}{-2\Delta w(w_o L_p C_p)}$$

$$\Rightarrow Z_{tank}(\Delta w) \approx \frac{j w_o L_p}{-2\Delta w(w_o L_p C_p)} = \boxed{-\frac{j}{2}\frac{1}{w_o C_p}\left(\frac{w_o}{\Delta w}\right)}$$

A Convenient Parameterization of LC Tank Impedance



$$Z_{tank}(\Delta w) \approx -\frac{j}{2}\frac{1}{w_o C_p}\left(\frac{w_o}{\Delta w}\right)$$

- Actual tank has loss that is modeled with R_p
  - Define Q according to actual tank

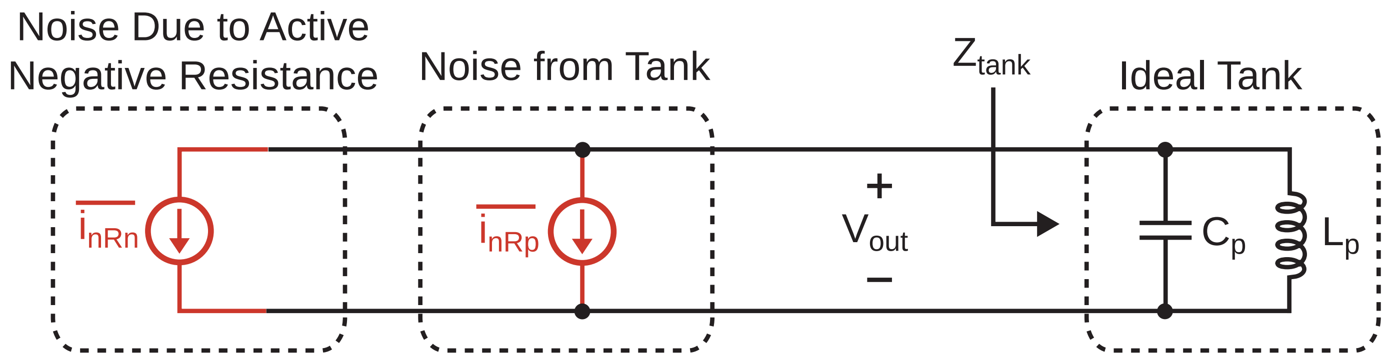
$$Q = R_p w_o C_p \quad \Rightarrow \quad \frac{1}{w_o C_p} = \frac{R_p}{Q}$$

- Parameterize ideal tank impedance in terms of Q of actual tank

$$Z_{tank}(\Delta w) \approx -\frac{j}{2}\frac{R_p}{Q}\left(\frac{w_o}{\Delta w}\right)$$

$$\Rightarrow \quad |Z_{tank}(\Delta f)|^2 \approx \boxed{\left(\frac{R_p}{2Q}\frac{f_o}{\Delta f}\right)^2}$$

Overall Noise Output Spectral Density



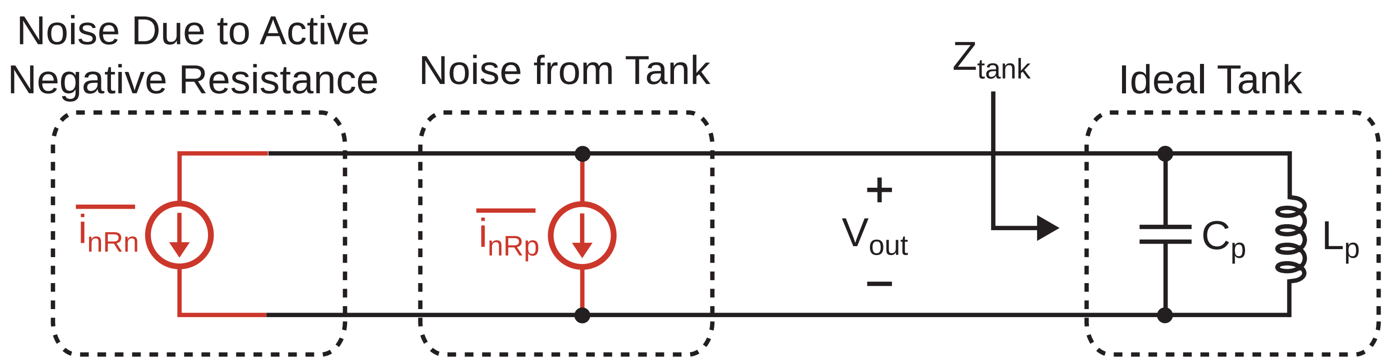
- **Assume noise from active negative resistance element and tank are uncorrelated**

$$\frac{\overline{v_{out}^2}}{\Delta f} = \left(\frac{\overline{i_{nRp}^2}}{\Delta f} + \frac{\overline{i_{nRn}^2}}{\Delta f}\right)|Z_{tank}(\Delta f)|^2$$

$$= \frac{\overline{i_{nRp}^2}}{\Delta f}\left(1 + \frac{\overline{i_{nRn}^2}}{\Delta f} \Big/ \frac{\overline{i_{nRp}^2}}{\Delta f}\right)|Z_{tank}(\Delta f)|^2$$

  - **Note that the above expression represents total noise that impacts both amplitude and phase of oscillator output**

Parameterize Noise Output Spectral Density



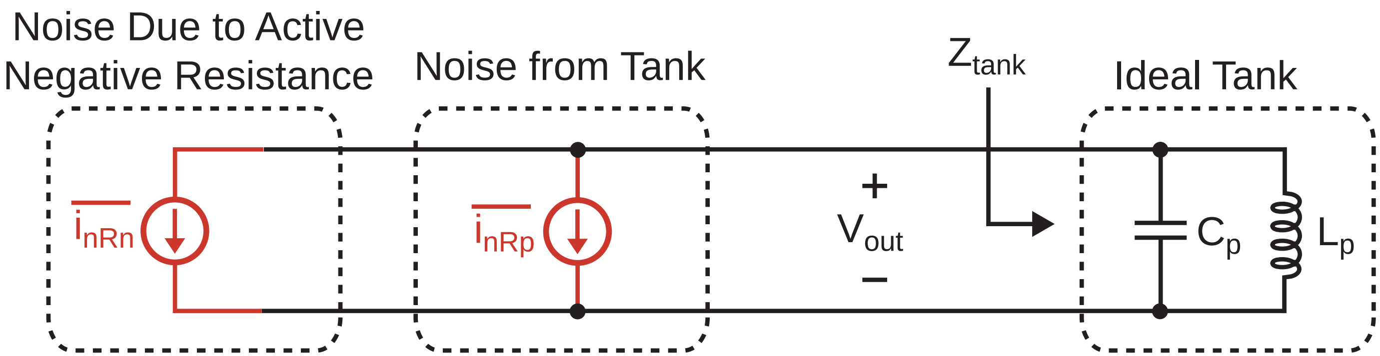
- **From previous slide**

$$\frac{\overline{v_{out}^2}}{\Delta f} = \frac{\overline{i_{nRp}^2}}{\Delta f}\underbrace{\left(1 + \frac{\overline{i_{nRn}^2}}{\Delta f} \Big/ \frac{\overline{i_{nRp}^2}}{\Delta f}\right)}_{F(\Delta f)} |Z_{tank}(\Delta f)|^2$$

- **F(Δf) is defined as**

$$F(\Delta f) = \frac{\text{total noise in tank at frequency } \Delta f}{\text{noise in tank due to tank loss at frequency } \Delta f}$$

Fill in Expressions



- **Noise from tank is due to resistor R_p**

$$\frac{\overline{i^2_{nRp}}}{\Delta f} = 4kT\frac{1}{R_p} \quad \text{(single-sided spectrum)}$$

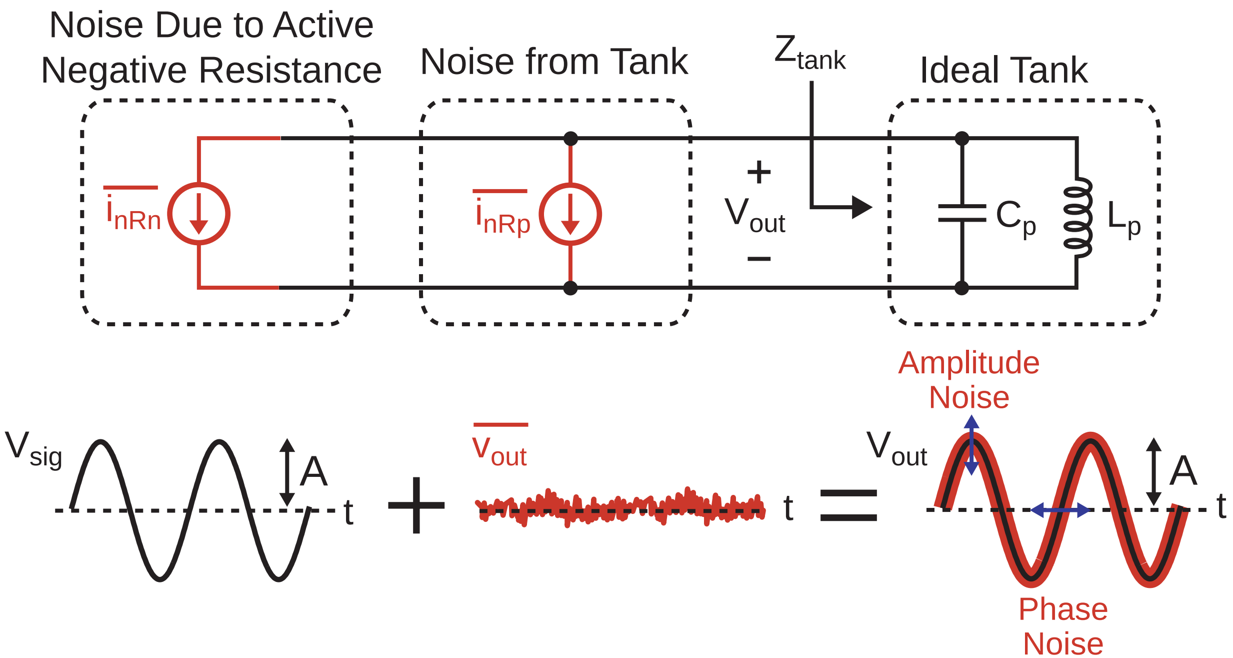
- **$Z_{tank}(\Delta f)$ found previously**

$$|Z_{tank}(\Delta f)|^2 \approx \left(\frac{R_p}{2Q}\frac{f_o}{\Delta f}\right)^2$$

- **Output noise spectral density expression (single-sided)**

$$\frac{\overline{v^2_{out}}}{\Delta f} = 4kT\frac{1}{R_p}F(\Delta f)\left(\frac{R_p}{2Q}\frac{f_o}{\Delta f}\right)^2 = \boxed{4kTF(\Delta f)R_p\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2}$$

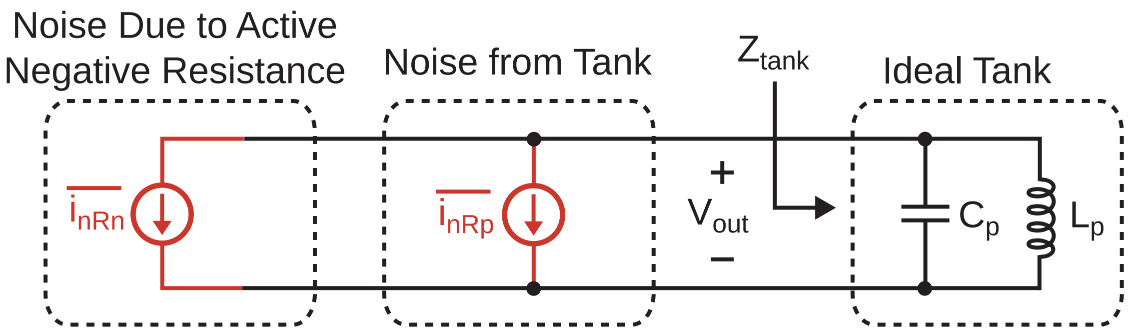
Separation into Amplitude and Phase Noise

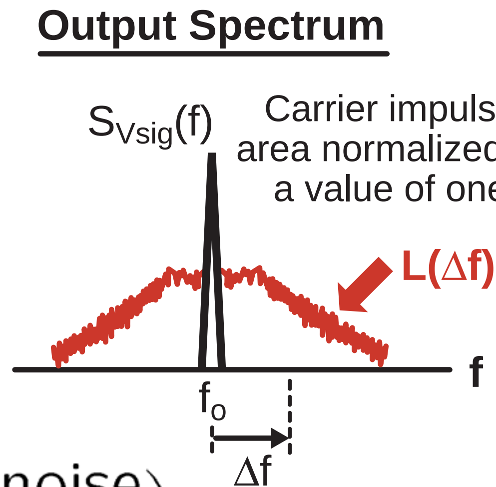


- **Equipartition theorem (see Tom Lee, p 534 (1st ed.)) states that noise impact splits evenly between amplitude and phase for V_{sig} being a sine wave**
  - **Amplitude variations suppressed by feedback in oscillator**

$$\Rightarrow \quad \frac{\overline{v_{out}^2}}{\Delta f}\Bigg|_{\text{phase}} = \boxed{2kTF(\Delta f)R_p\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2} \quad \text{(single-sided)}$$

Output Phase Noise Spectrum (Leeson's Formula)





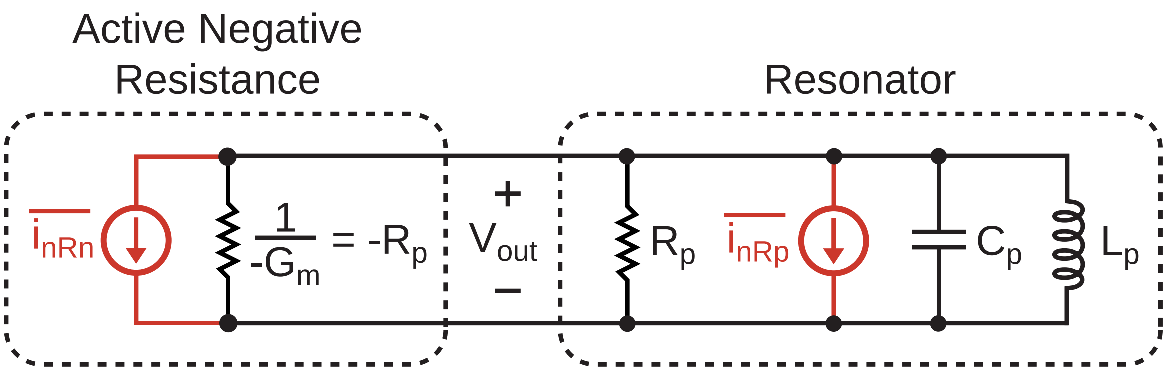
$$L(\Delta f) = 10\log\left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}}\right)$$

- **All power calculations are referenced to the tank loss resistance, R_p**

$$P_{sig} = \frac{V_{sig,rms}^2}{R_p} = \frac{(A/\sqrt{2})^2}{R_p}, \quad S_{noise}(\Delta f) = \frac{1}{R_p}\frac{\overline{v_{out}^2}}{\Delta f}$$

$$L(\Delta f) = 10\log\left(\frac{S_{noise}(\Delta f)}{P_{sig}}\right) = 10\log\left(\frac{2kTF(\Delta f)}{P_{sig}}\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)$$

Example: Active Noise Same as Tank Noise

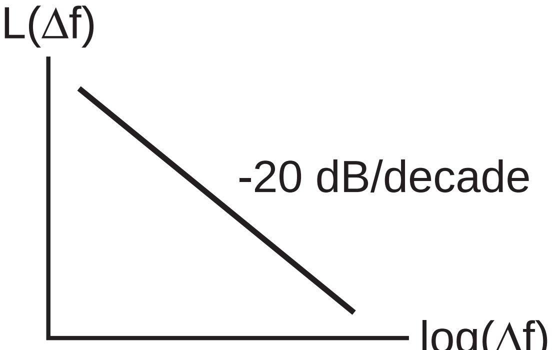


- Noise factor for oscillator in this case is

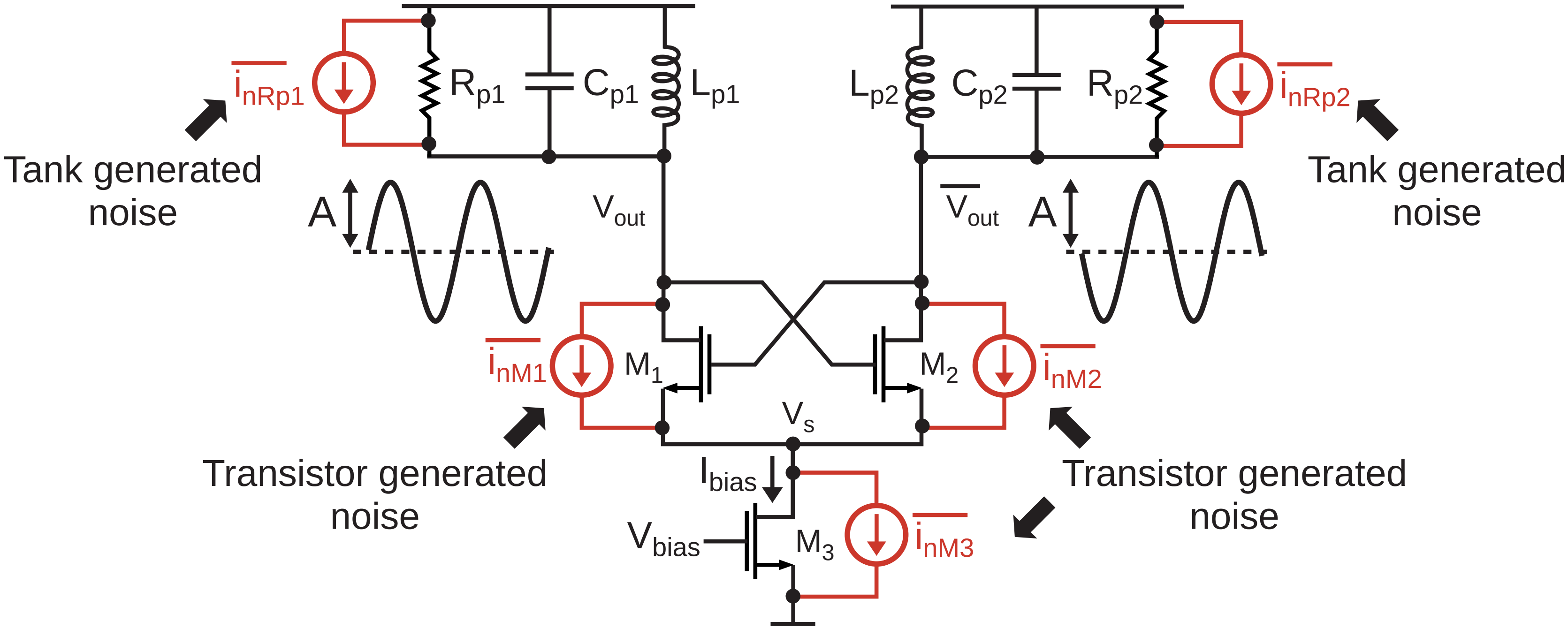
$$F(\Delta f) = 1 + \frac{\overline{i^2_{nRn}}}{\Delta f} \Big/ \frac{\overline{i^2_{nRp}}}{\Delta f} = 2$$

- Resulting phase noise

$$L(\Delta f) = 10 \log \left(\frac{4kT}{P_{sig}} \left(\frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \right)$$

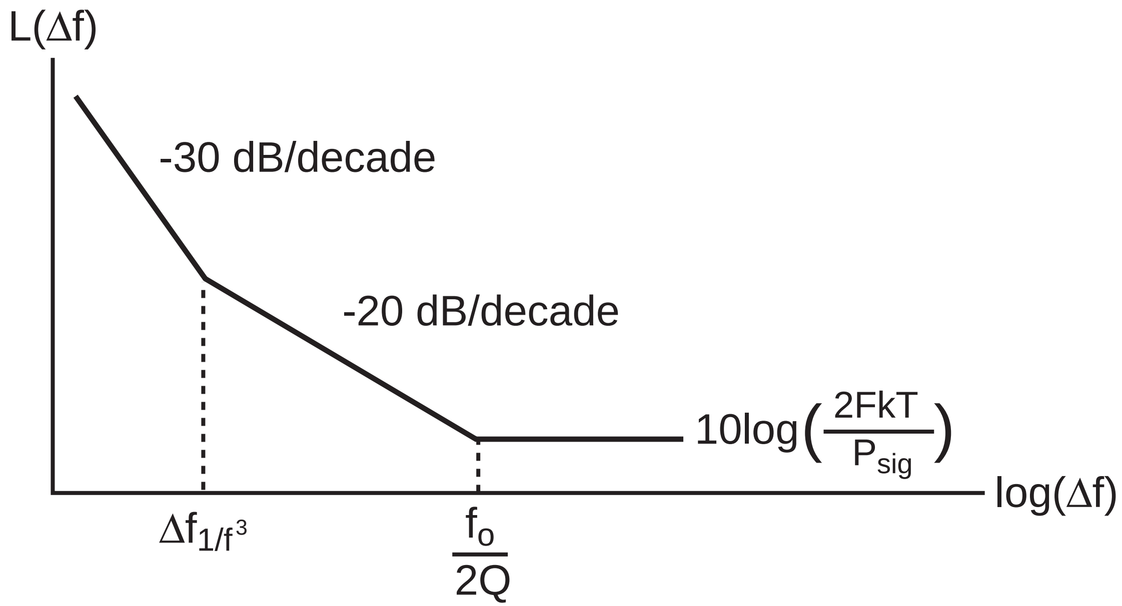


The Actual Situation is Much More Complicated



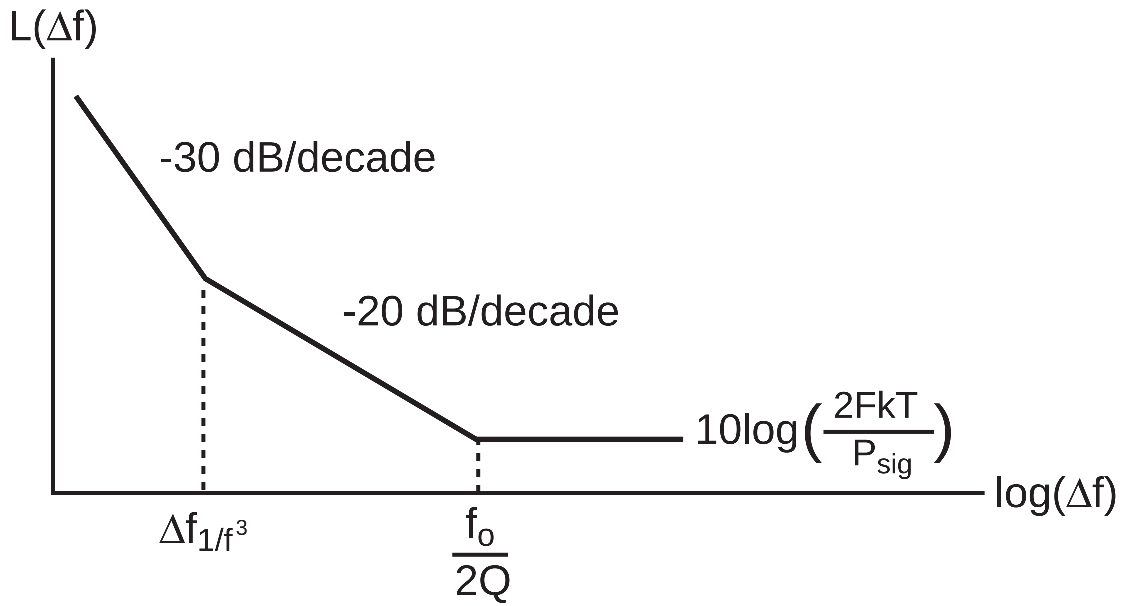
- **Impact of tank generated noise easy to assess**
- **Impact of transistor generated noise is complicated**
  - **Noise from M_1 and M_2 is modulated on and off**
  - **Noise from M_3 is modulated before influencing V_{out}**
  - **Transistors have 1/f noise**
- **Also, transistors can degrade Q of tank**

Phase Noise of A Practical Oscillator



- **Phase noise drops at -20 dB/decade over a wide frequency range, but deviates from this at:**
  - **Low frequencies – slope increases (often -30 dB/decade)**
  - **High frequencies – slope flattens out (oscillator tank does not filter all noise sources)**
- **Frequency breakpoints and magnitude scaling are not readily predicted by the analysis approach taken so far**

Phase Noise of A Practical Oscillator

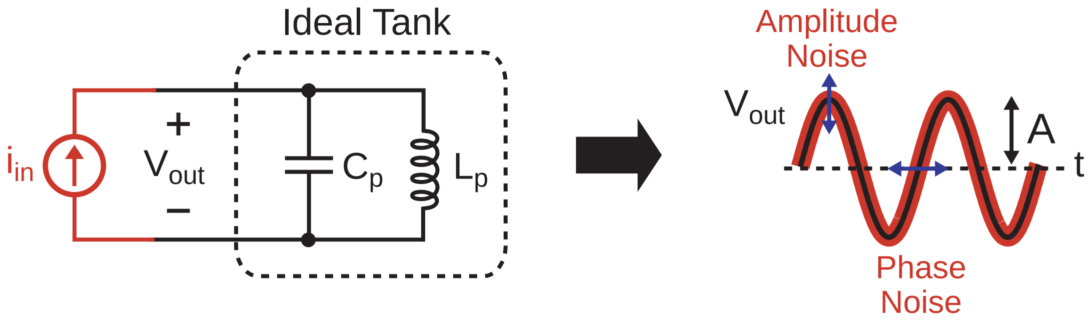


- **Leeson proposed an ad hoc modification of the phase noise expression to capture the above noise profile**

$$L(\Delta f) = 10\log\left(\frac{2FkT}{P_{sig}}\left(1+\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)\left(1+\frac{\Delta f_{1/f^3}}{|\Delta f|}\right)\right)$$

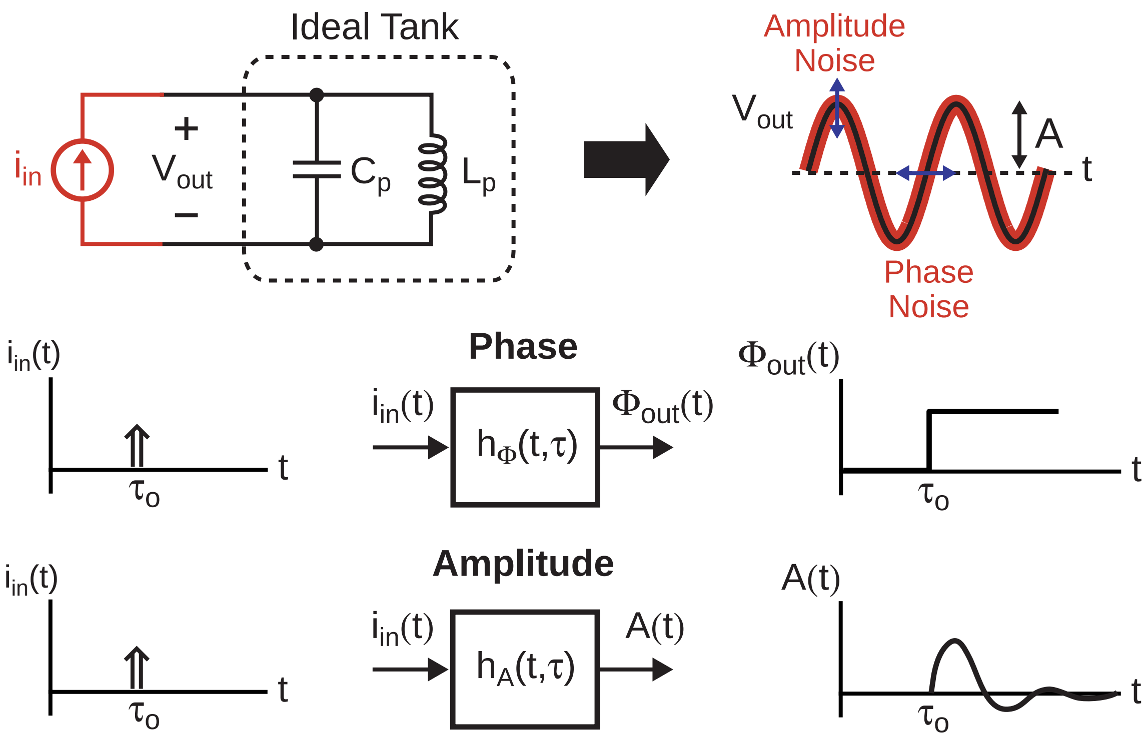
  - **Note: he assumed that F(Δf) was constant over frequency**

A More Sophisticated Analysis Method



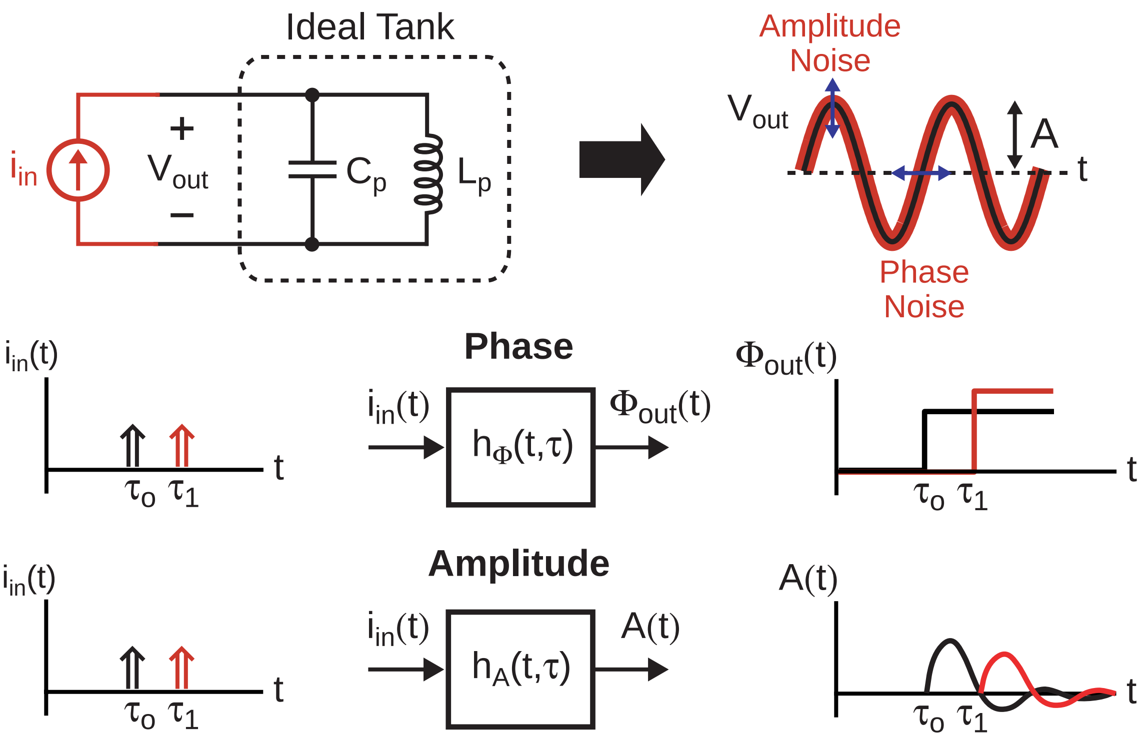
- **Our concern is what happens when noise current produces a voltage across the tank**
  - **Such voltage deviations give rise to both amplitude and phase noise**
  - **Amplitude noise is suppressed through feedback (or by amplitude limiting in following buffer stages)**
    - Our main concern is phase noise
- **We argued that impact of noise divides equally between amplitude and phase for sine wave outputs**
  - **What happens when we have a non-sine wave output?**

Modeling of Phase and Amplitude Perturbations



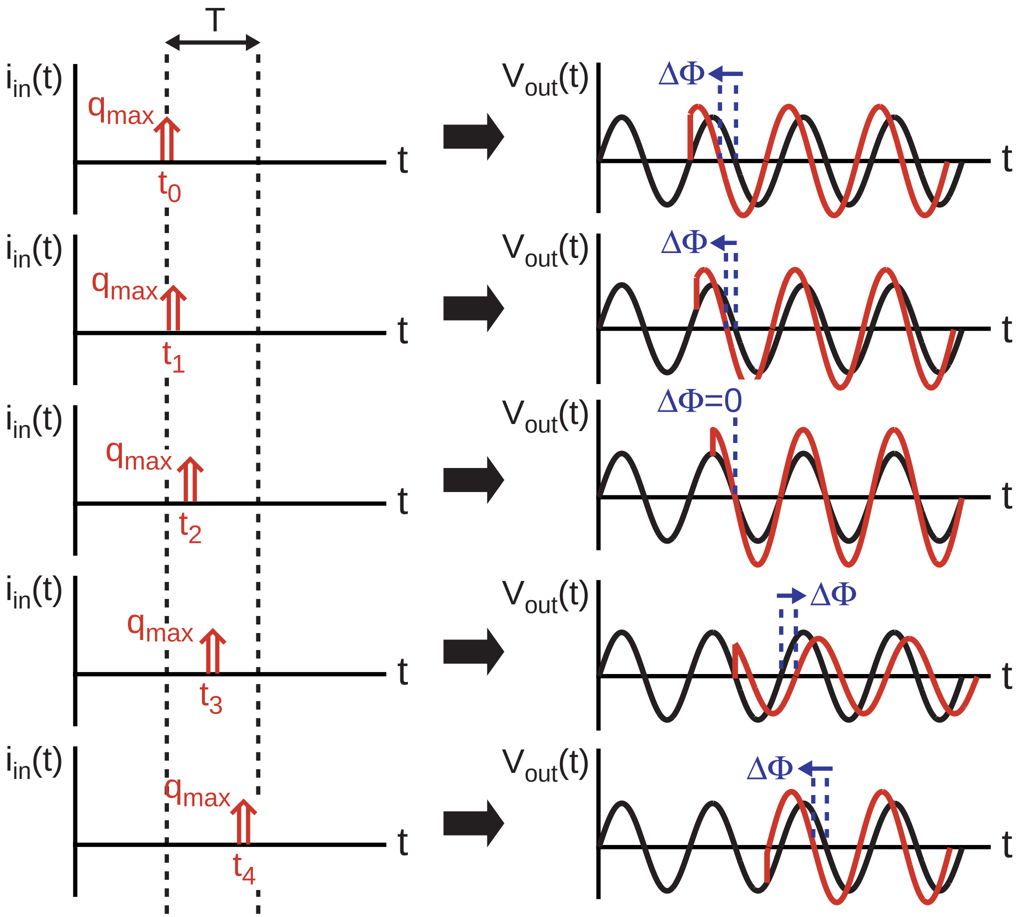
- **Characterize impact of current noise on amplitude and phase through their associated impulse responses**
  - **Phase deviations are accumulated**
  - **Amplitude deviations are suppressed**

Impact of Noise Current is Time-Varying



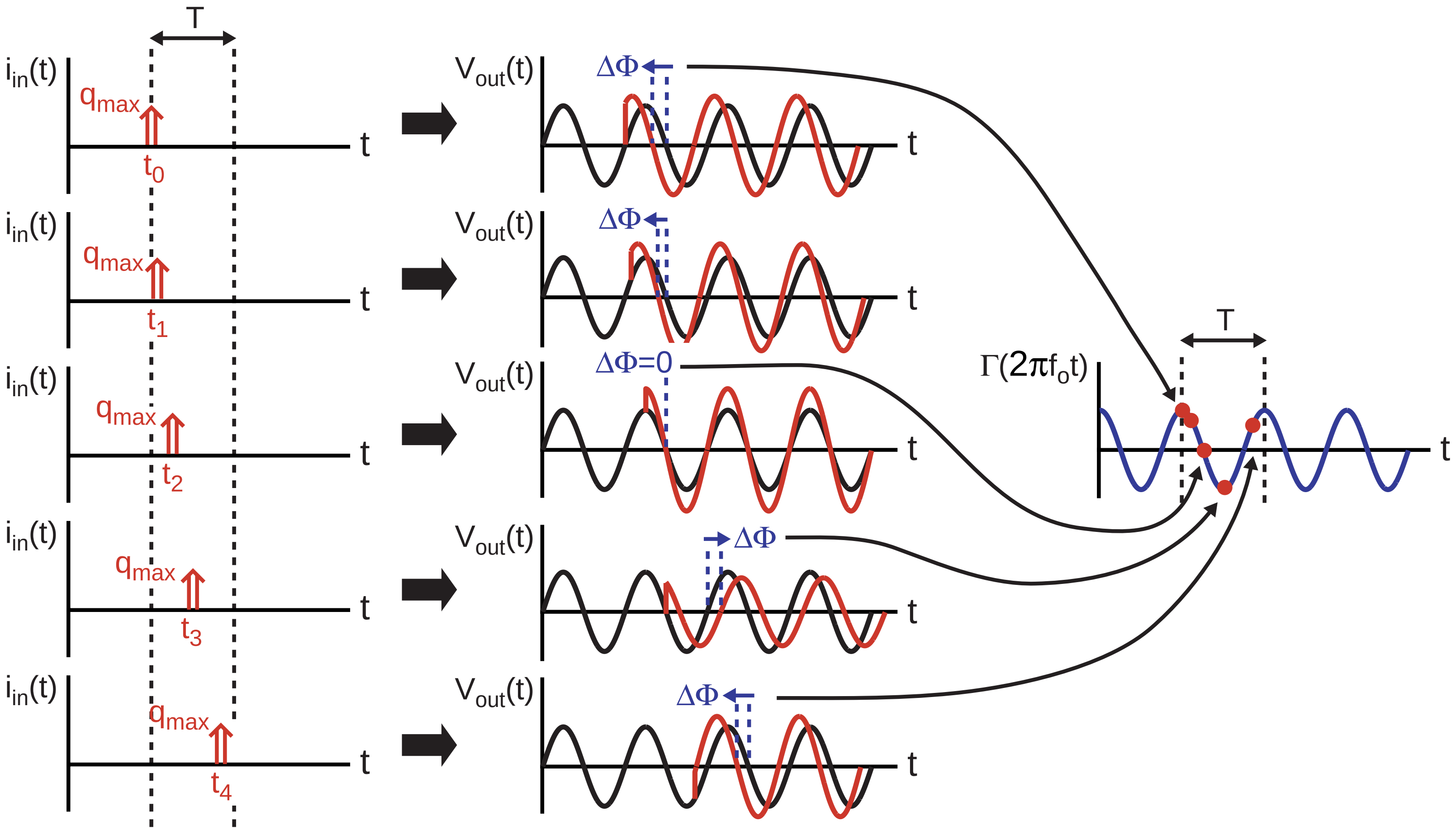
- **If we vary the time at which the current impulse is injected, its impact on phase and amplitude changes**
  - **Need a time-varying model**

Illustration of Time-Varying Impact of Noise on Phase



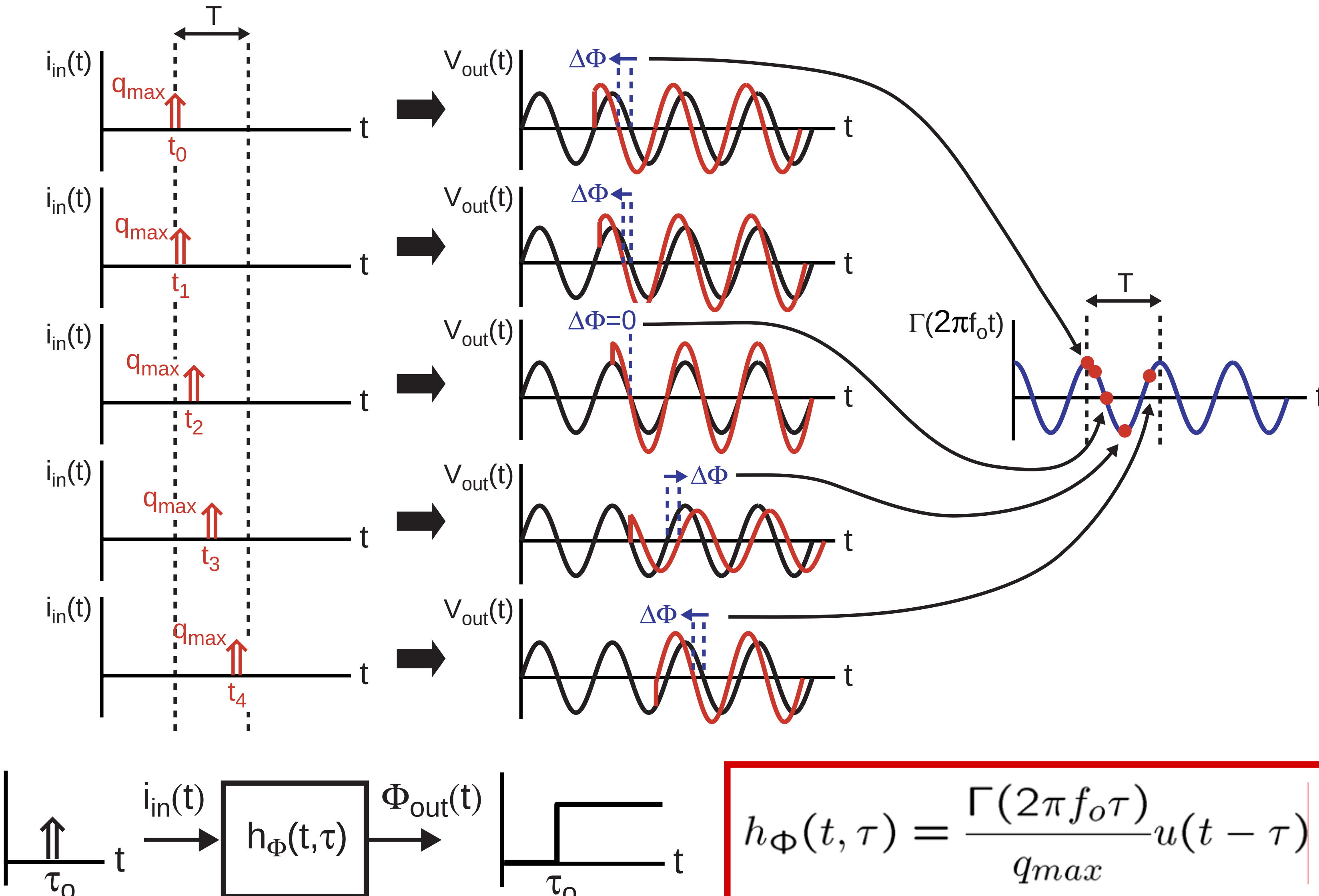
- **High impact on phase when impulse occurs close to the zero crossing of the VCO output**
- **Low impact on phase when impulse occurs at peak of output**

Define Impulse Sensitivity Function (ISF) – $\Gamma(2\pi f_o t)$



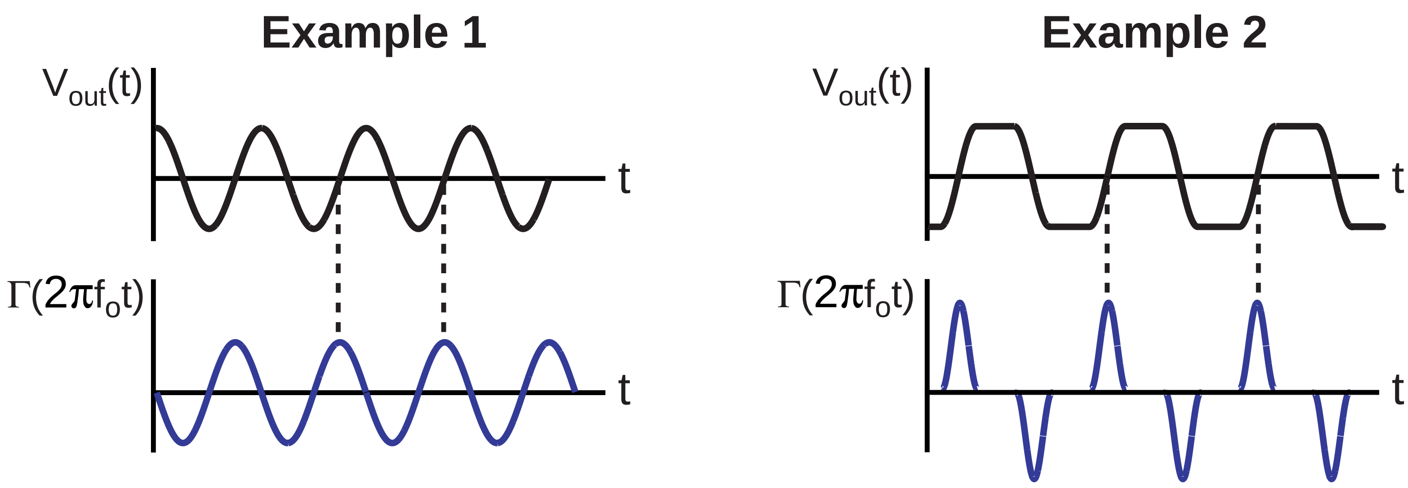
- ISF constructed by calculating phase deviations as impulse position is varied
  - Observe that it is periodic with same period as VCO output

Parameterize Phase Impulse Response in Terms of ISF



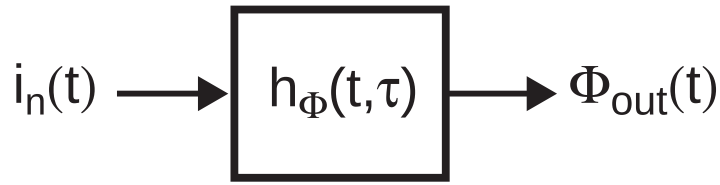
$$h_\Phi(t,\tau) = \frac{\Gamma(2\pi f_o \tau)}{q_{max}} u(t-\tau)$$

Examples of ISF for Different VCO Output Waveforms



- **ISF (i.e., Γ) is approximately proportional to derivative of VCO output waveform**
  - **Its magnitude indicates where VCO waveform is most sensitive to noise current into tank with respect to creating phase noise**
- **ISF is periodic**
- **In practice, derive it from simulation of the VCO**

Phase Noise Analysis Using LTV Framework



- **Computation of phase deviation for an arbitrary noise current input**

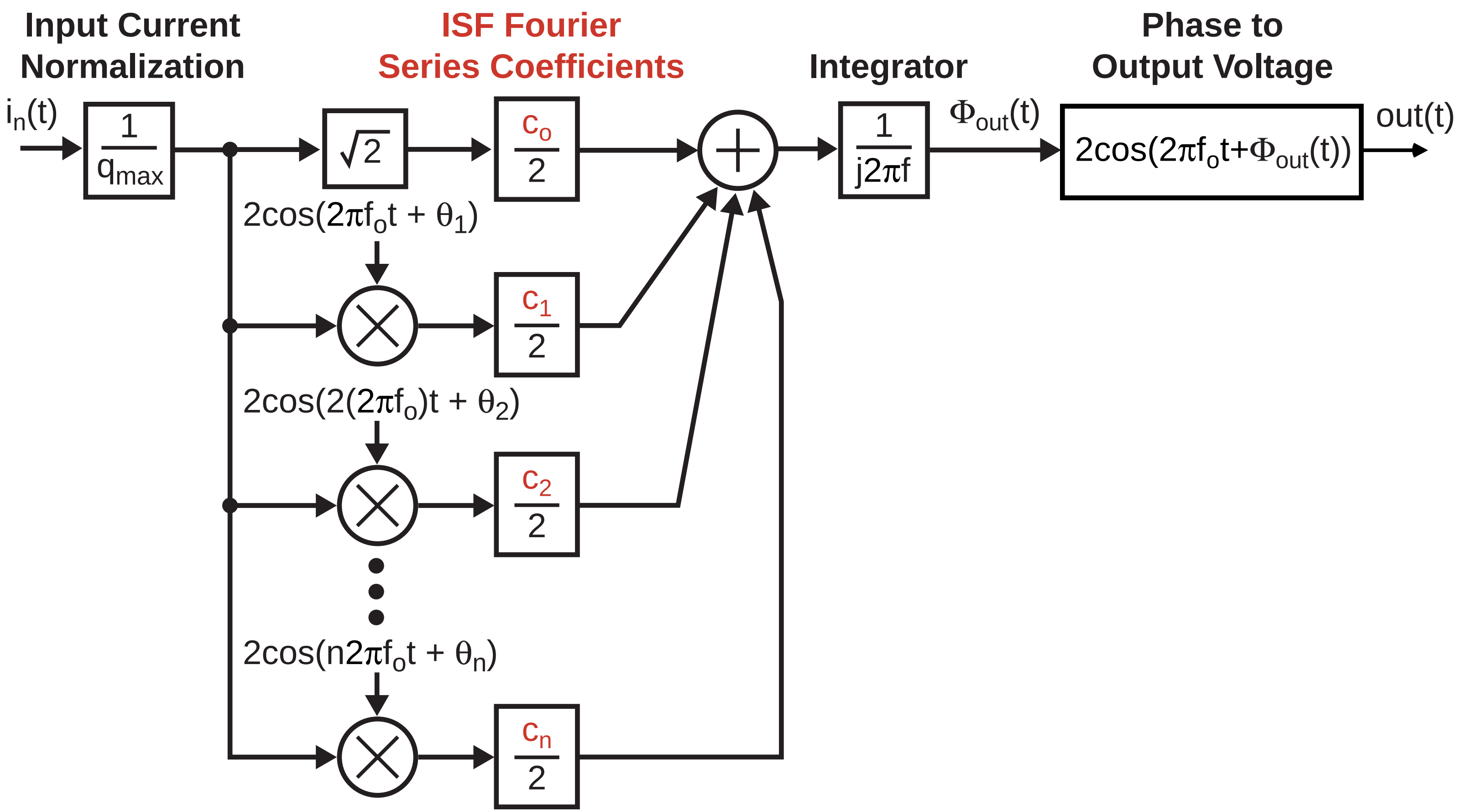
$$\Phi_{out}(t) = \int_{-\infty}^{\infty} h_\Phi(t,\tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(2\pi f_o \tau) i_n(\tau) d\tau$$

- **Analysis simplified if we describe ISF in terms of its Fourier series (note: c_o here is different than book)**

$$\Gamma(2\pi f_o \tau) = \frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n 2\pi f_o \tau + \theta_n)$$

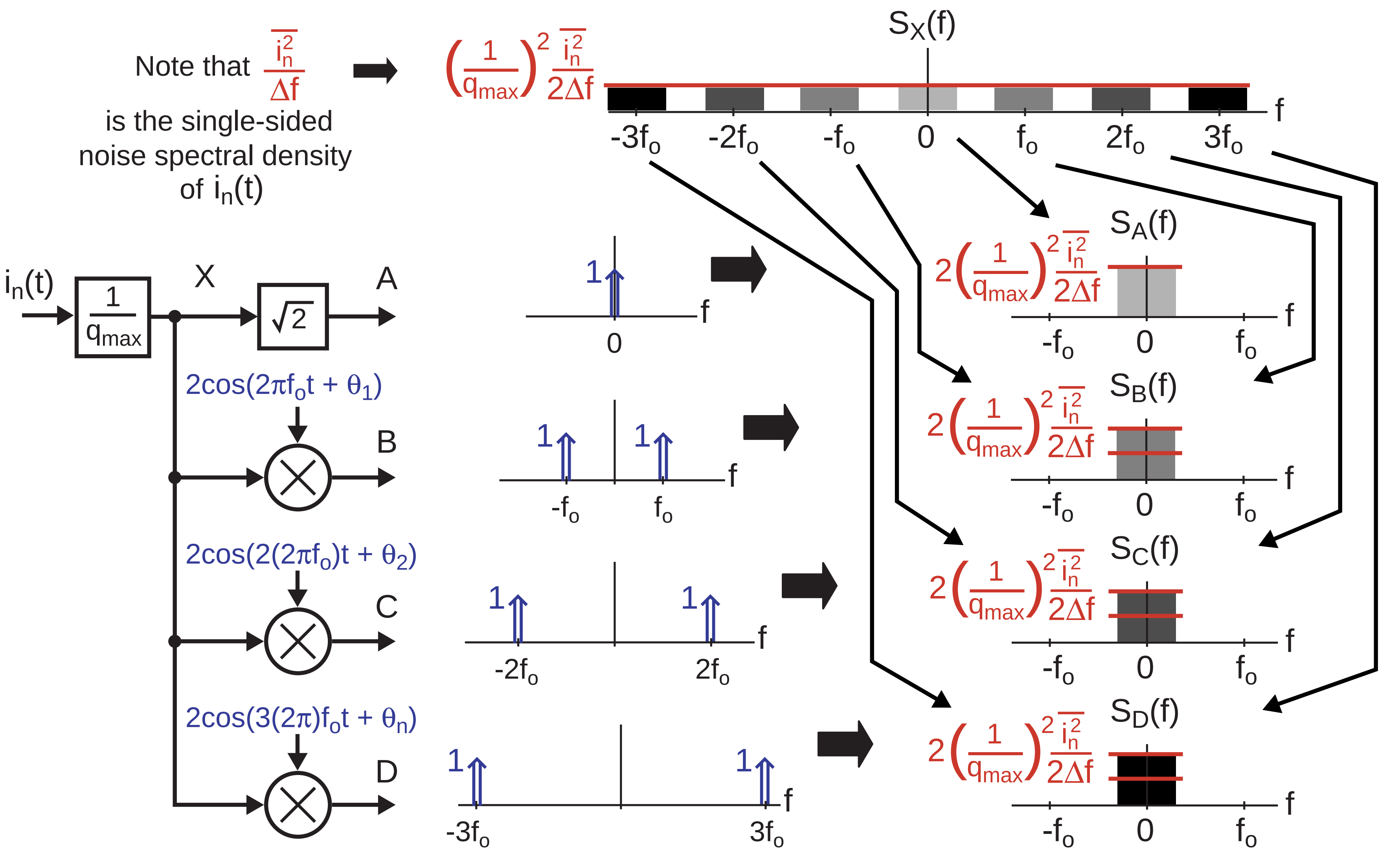
$$\Rightarrow \boxed{\Phi_{out}(t) = \int_{-\infty}^{t} \left(\frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n 2\pi f_o \tau + \theta_n) \right) \frac{i_n(\tau)}{q_{max}} d\tau}$$

Block Diagram of LTV Phase Noise Expression

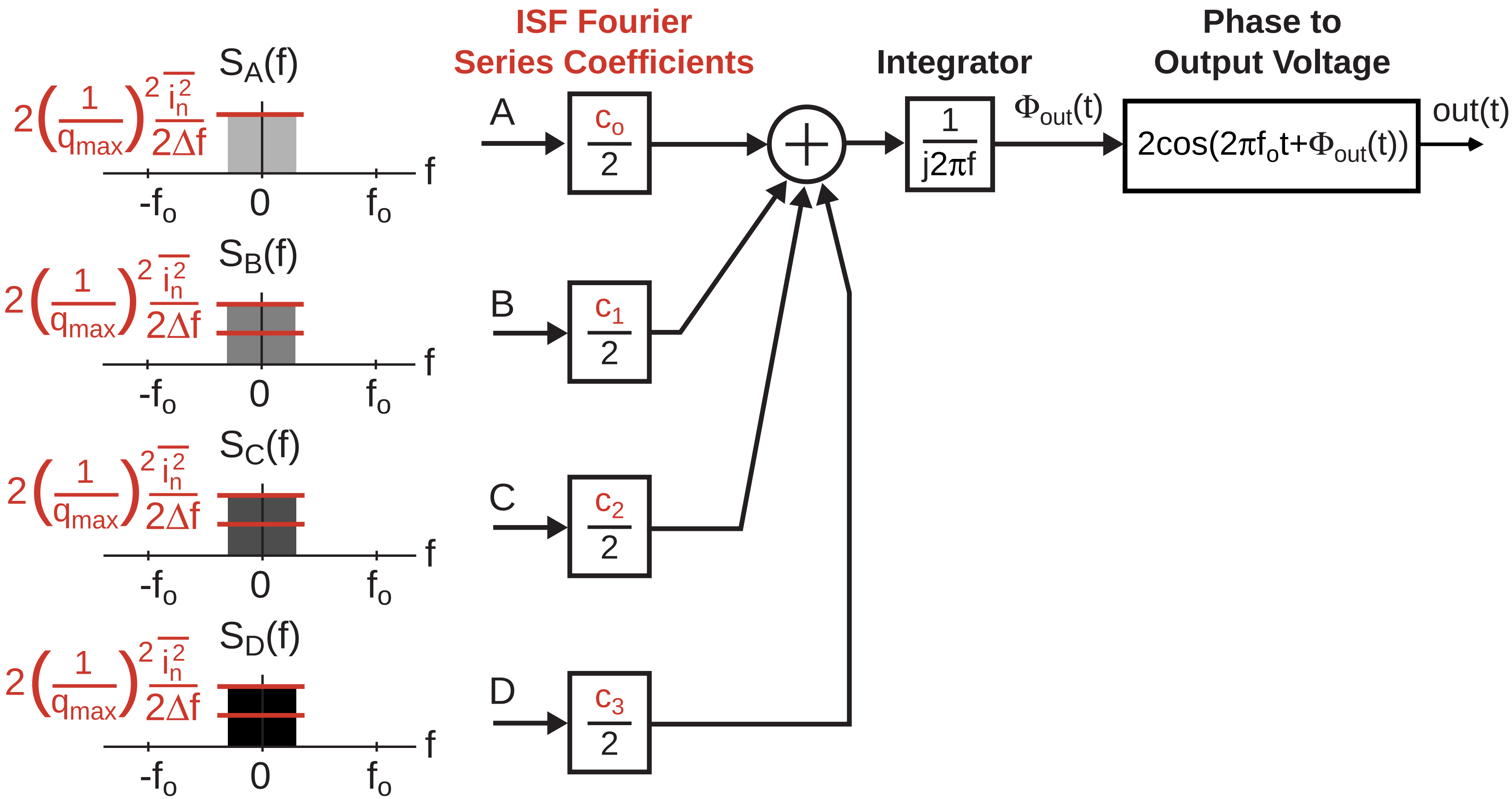


- Noise from current source is mixed down from different frequency bands and scaled according to ISF coefficients

Phase Noise Calculation for White Noise Input (Part 1)



Phase Noise Calculation for White Noise Input (Part 2)



$$S_{\Phi_{out}}(f) = \left|\frac{1}{j2\pi f}\right|^2 \left(\left(\frac{c_o}{2}\right)^2 S_A(f) + \left(\frac{c_1}{2}\right)^2 S_B(f) + \cdots\right)$$

Spectral Density of Phase Signal

- **From the previous slide**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\left(\frac{c_o}{2}\right)^2 S_A(f) + \left(\frac{c_1}{2}\right)^2 S_B(f) + \cdots\right)$$

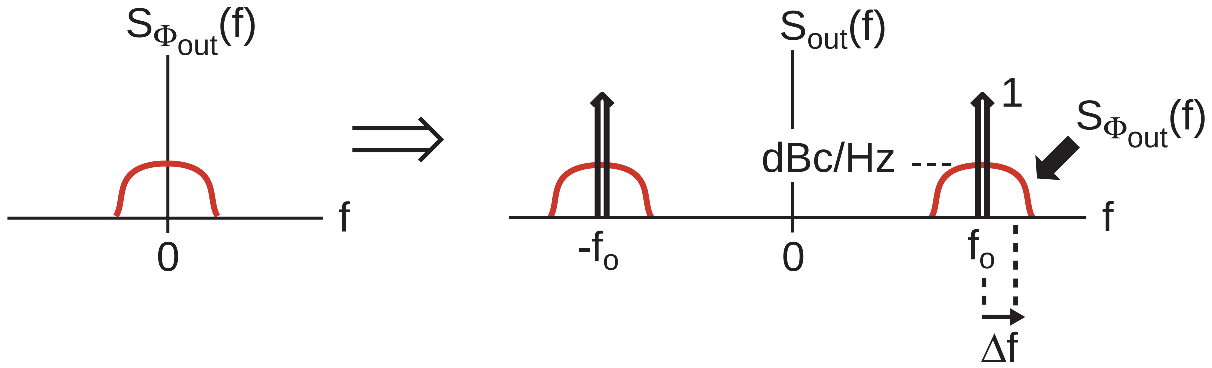
- **Substitute in for $S_A(f)$, $S_B(f)$, etc.**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\left(\frac{c_o}{2}\right)^2 + \left(\frac{c_1}{2}\right)^2 + \cdots\right) 2 \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{2\Delta f}$$

- **Resulting expression**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}$$

Output Phase Noise



- **We now know**

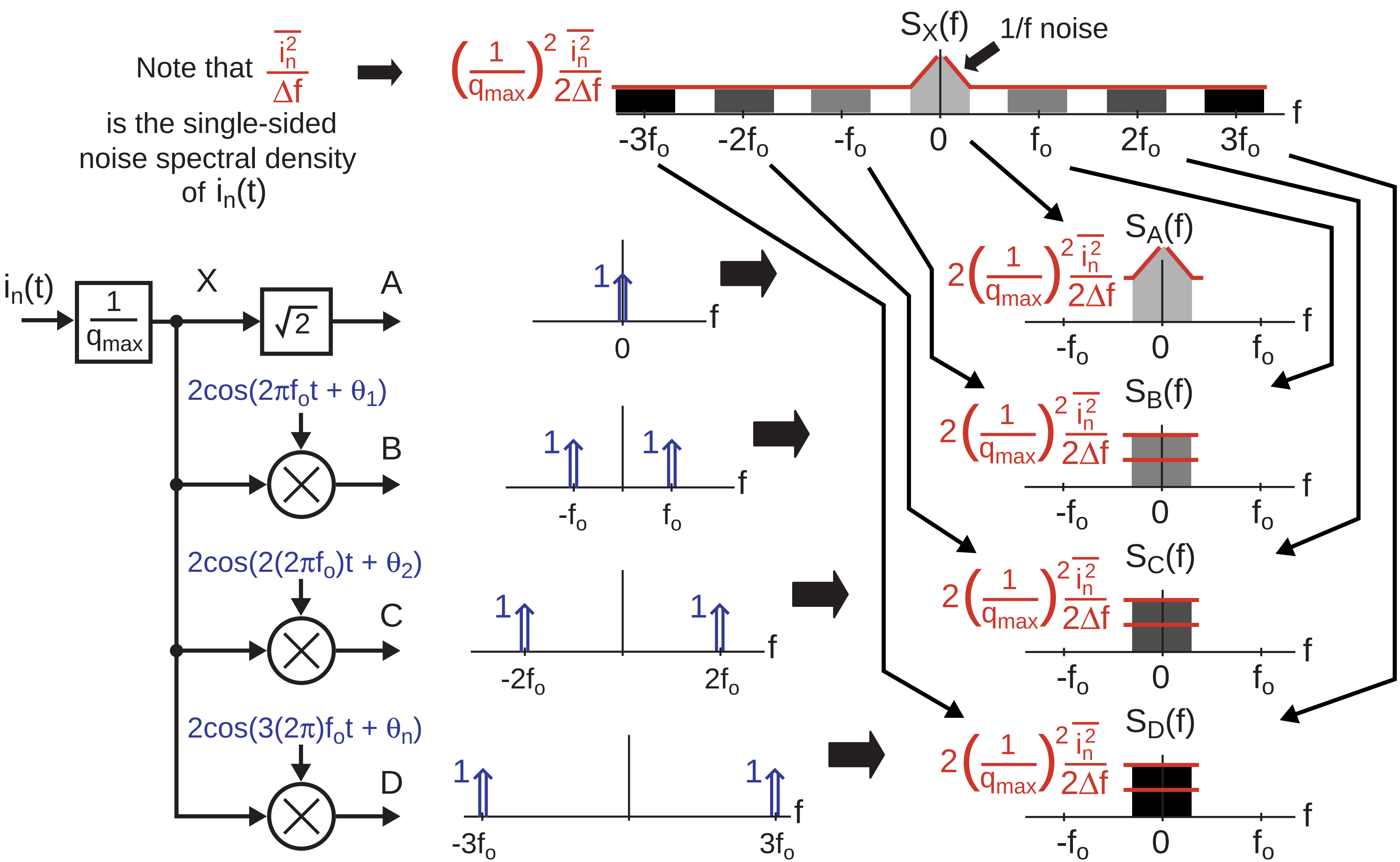
$$S_{\Phi_{out}}(f) = \left|\frac{1}{2\pi f}\right|^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}$$

$$L(\Delta f) = 10\log(S_{\Phi_{out}}(\Delta f))$$

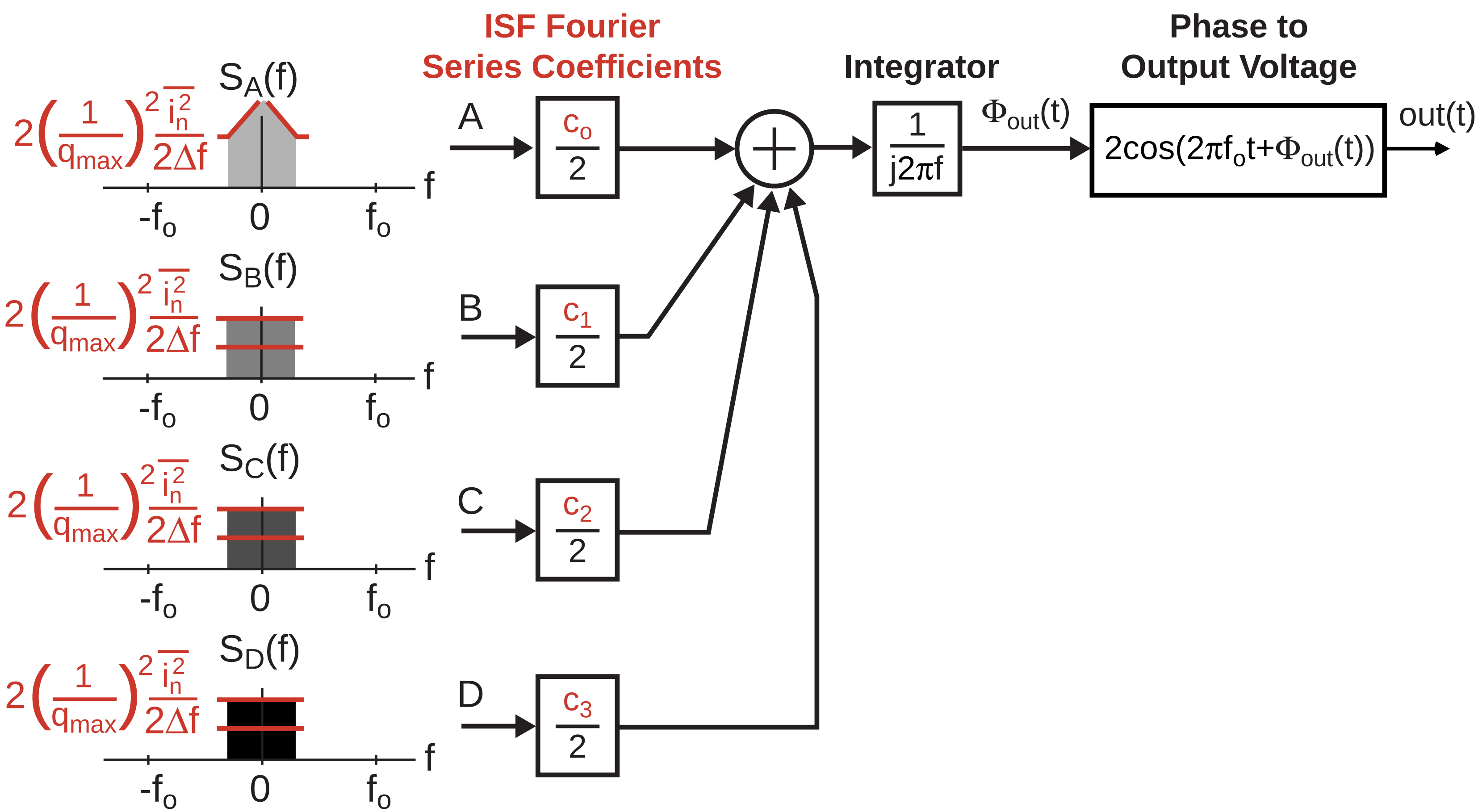
- **Resulting phase noise**

$$L(\Delta f) = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\right)$$

The Impact of 1/f Noise in Input Current (Part 1)



The Impact of 1/f Noise in Input Current (Part 2)



$$S_{\Phi_{out}}(f)\Big|_{1/f^3} = \left|\frac{1}{j2\pi f}\right|^2 \left(\frac{c_o}{2}\right)^2 S_A(f)$$

Calculation of Output Phase Noise in 1/f³ region

- **From the previous slide**

$$S_{\Phi_{out}}(f)\Big|_{1/f^3} = \left(\frac{1}{2\pi f}\right)^2 \left(\frac{c_o}{2}\right)^2 S_A(f)$$

- **Assume that input current has 1/f noise with corner frequency $f_{1/f}$**

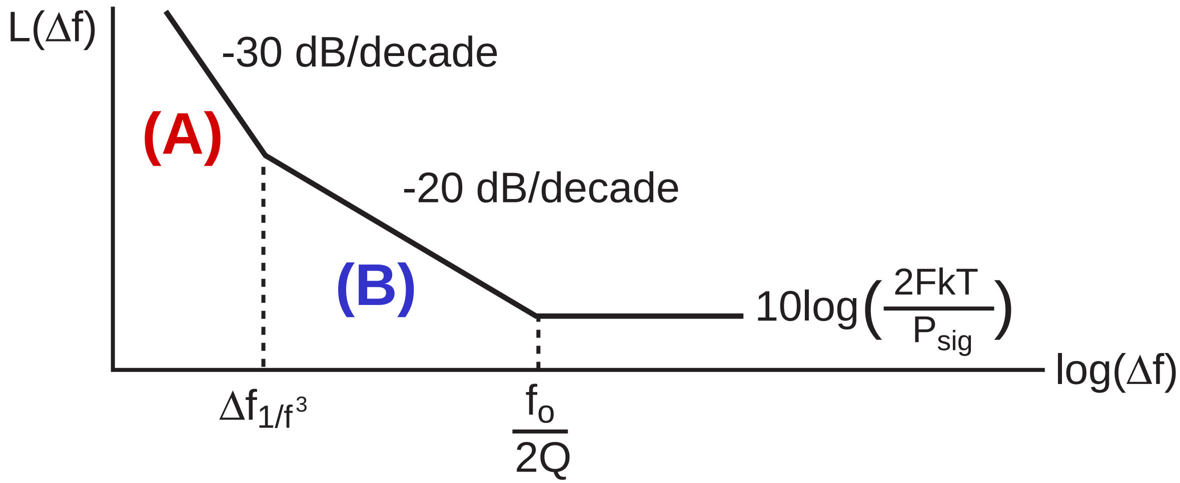
$$S_A(f) = \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\left(\frac{f_{1/f}}{\Delta f}\right)$$

- **Corresponding output phase noise**

$$L(\Delta f)\Big|_{1/f^3} = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2\left(\frac{c_o}{2}\right)^2 S_A(f)\right)$$

$$= 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(c_o^2\right)\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\left(\frac{f_{1/f}}{\Delta f}\right)\right)$$

Calculation of 1/f³ Corner Frequency



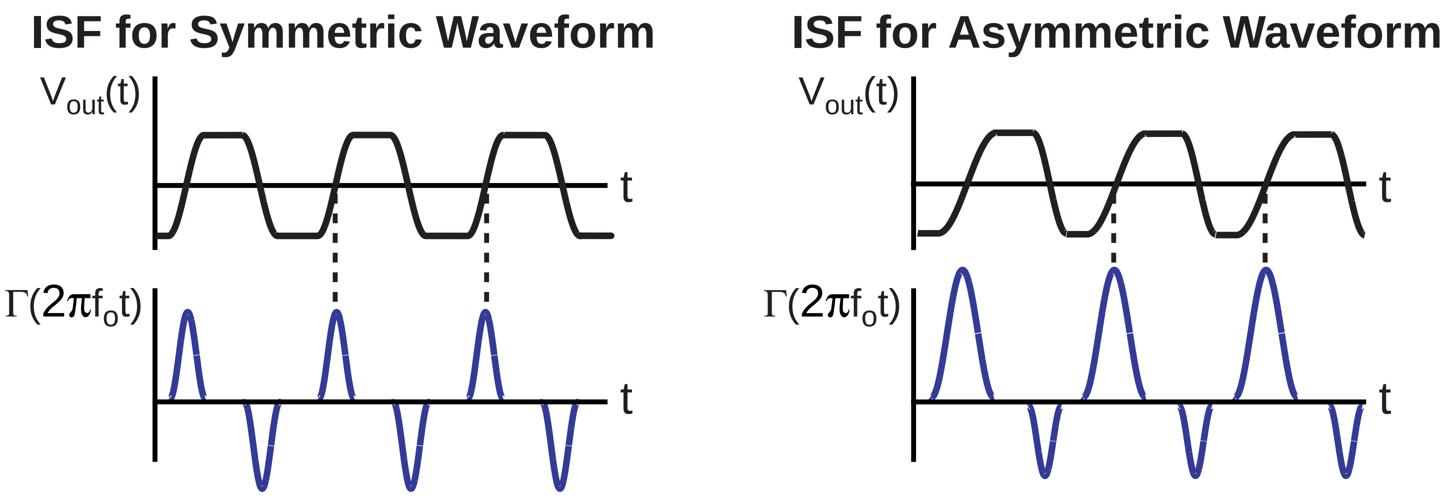
(A) $$L(\Delta f)\Big|_{1/f^3} = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(c_o^2\right)\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\left(\frac{f_{1/f}}{\Delta f}\right)\right)$$

(B) $$L(\Delta f) = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right)\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\right)$$

(A) = (B) at:

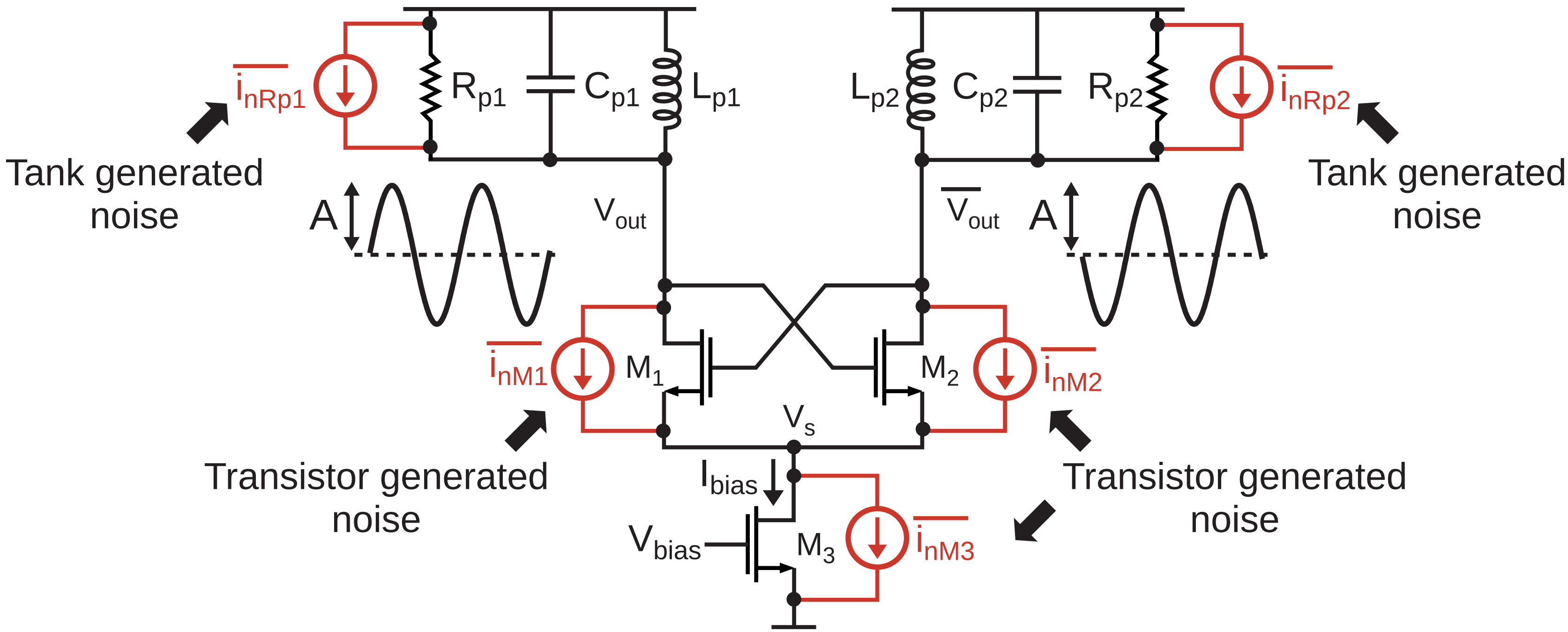
$$\Rightarrow \quad \Delta f_{1/f^3} = \left(c_o^2 \Big/ \sum_{n=0}^{\infty} c_n^2\right) f_{1/f}$$

Impact of Oscillator Waveform on 1/f³ Phase Noise



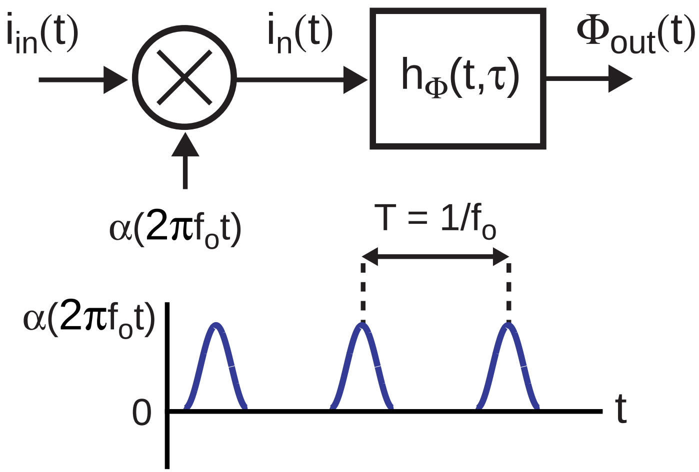
- **Key Fourier series coefficient of ISF for 1/f³ noise is c_o**
  - **If DC value of ISF is zero, c_o is also zero**
- **For symmetric oscillator output waveform**
  - **DC value of ISF is zero – no upconversion of flicker noise! (i.e. output phase noise does not have 1/f³ region)**
- **For asymmetric oscillator output waveform**
  - **DC value of ISF is nonzero – flicker noise has impact**

Issue – We Have Ignored Modulation of Current Noise



- **In practice, transistor generated noise is modulated by the varying bias conditions of its associated transistor**
  - **As transistor goes from saturation to triode to cutoff, its associated noise changes dramatically**
- **Can we include this issue in the LTV framework?**

Inclusion of Current Noise Modulation



- **Recall**

$$\Phi_{out}(t) = \int_{-\infty}^{\infty} h_{\Phi}(t,\tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(2\pi f_o \tau) i_n(\tau) d\tau$$

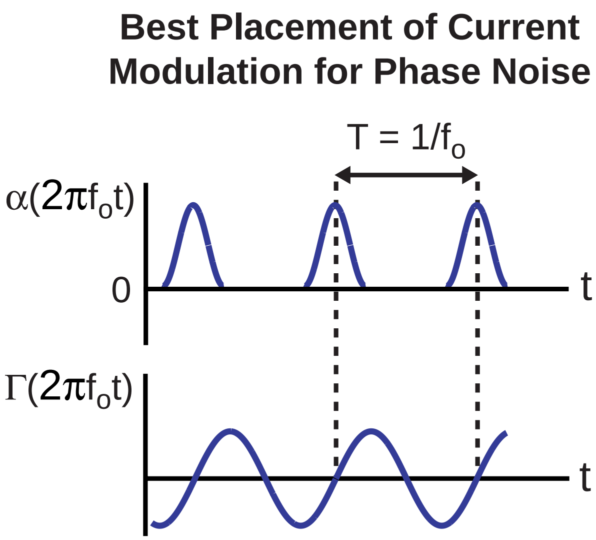
- **By inspection of figure**

$$\Rightarrow \quad \Phi_{out}(t) = \frac{1}{q_{max}} \int_{-\infty}^{t} \underline{\Gamma(2\pi f_o \tau)\alpha(2\pi f_o \tau)} i_{in}(\tau) d\tau$$

- **We therefore apply previous framework with ISF as**

$$\boxed{\Gamma_{eff}(2\pi f_o \tau) = \Gamma(2\pi f_o \tau)\alpha(2\pi f_o \tau)}$$

Placement of Current Modulation for Best Phase Noise

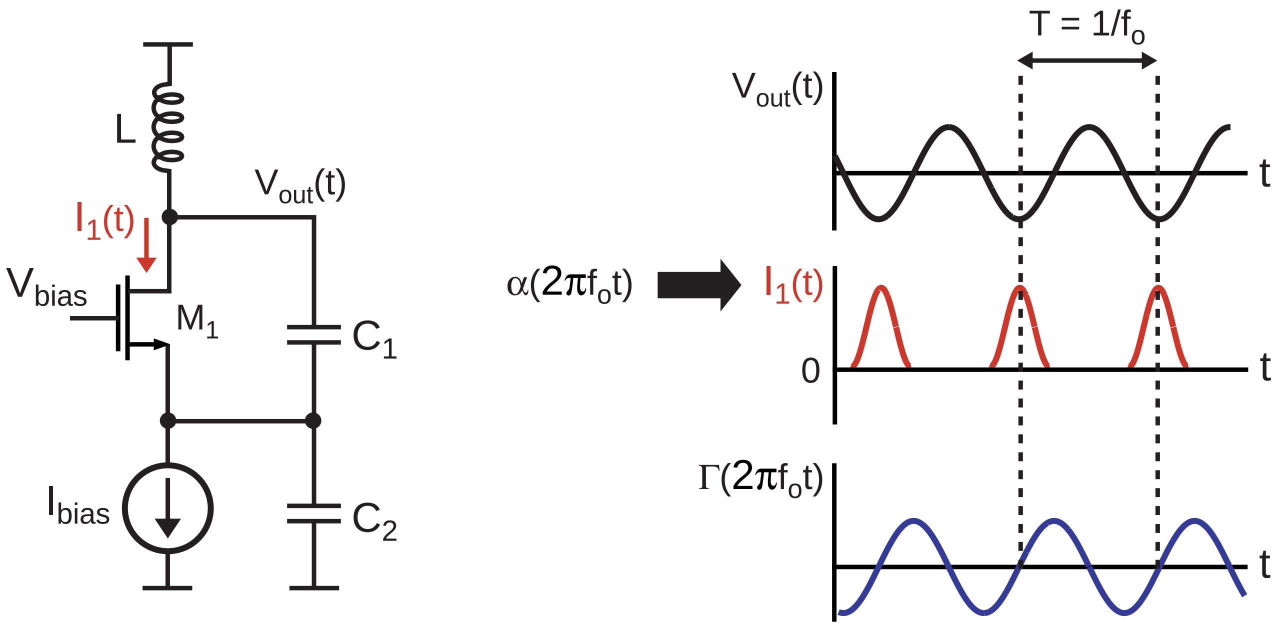


- **Phase noise expression (ignoring 1/f noise)**

$$L(\Delta f) = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\right)$$

- **Minimum phase noise achieved by minimizing sum of square of Fourier series coefficients (i.e. rms value of Γ_{eff})**

Colpitts Oscillator Provides Optimal Placement of α

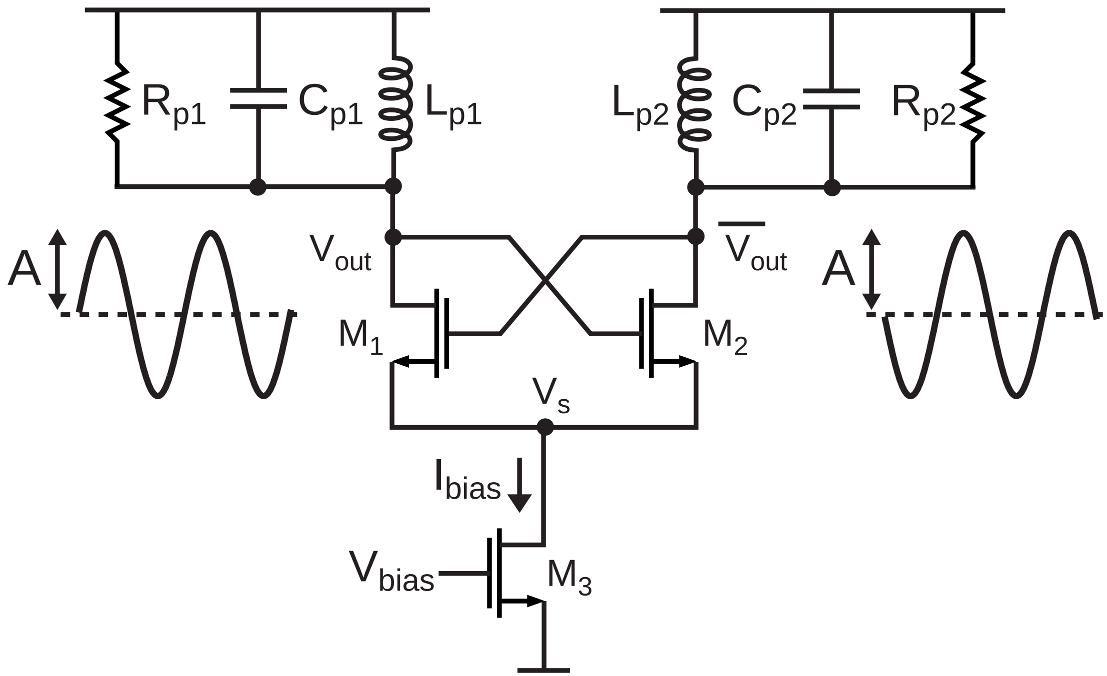


- **Current is injected into tank at bottom portion of VCO swing**
  - **Current noise accompanying current has minimal impact on VCO output phase**

Summary of LTV Phase Noise Analysis Method

- **Step 1: calculate the impulse sensitivity function of each oscillator noise source using a simulator**
- **Step 2: calculate the noise current modulation waveform for each oscillator noise source using a simulator**
- **Step 3: combine above results to obtain $\Gamma_{eff}(2\pi f_o t)$ for each oscillator noise source**
- **Step 4: calculate Fourier series coefficients for each $\Gamma_{eff}(2\pi f_o t)$**
- **Step 5: calculate spectral density of each oscillator noise source (before modulation)**
- **Step 6: calculate overall output phase noise using the results from Step 4 and 5 and the phase noise expressions derived in this lecture (or the book)**

Alternate Approach for Negative Resistance Oscillator

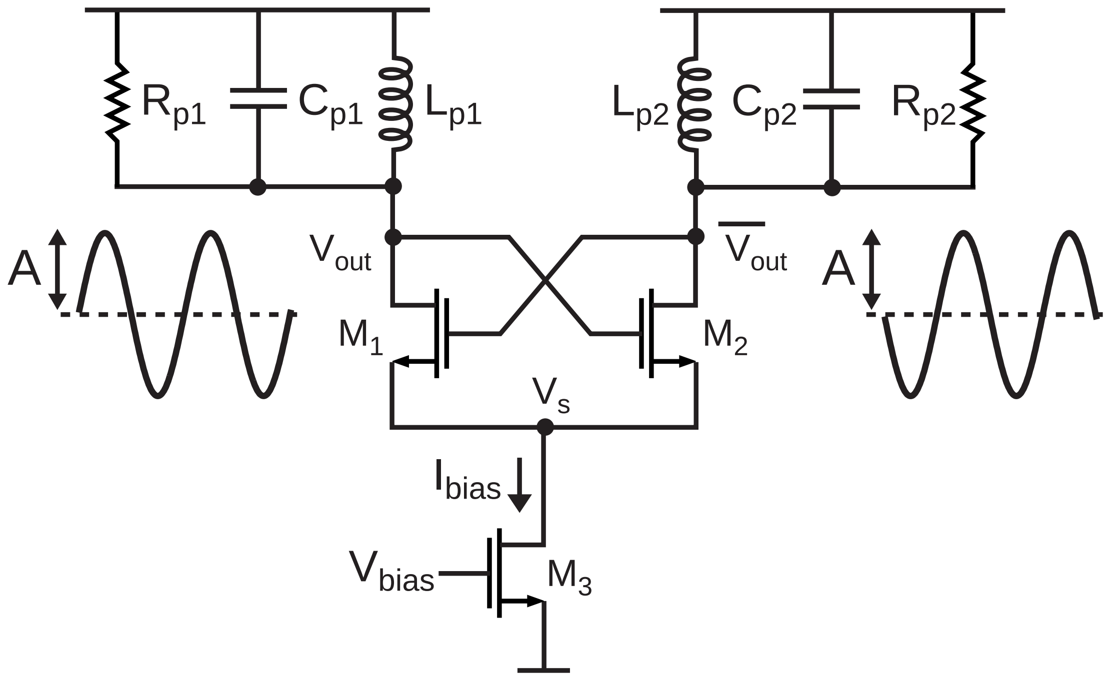


- **Recall Leeson's formula**

$$L(\Delta f) = 10\log\left(\frac{2kTF(\Delta f)}{P_{sig}}\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)$$

  - **Key question: how do you determine F(Δf)?**

F(Δf) Has Been Determined for This Topology



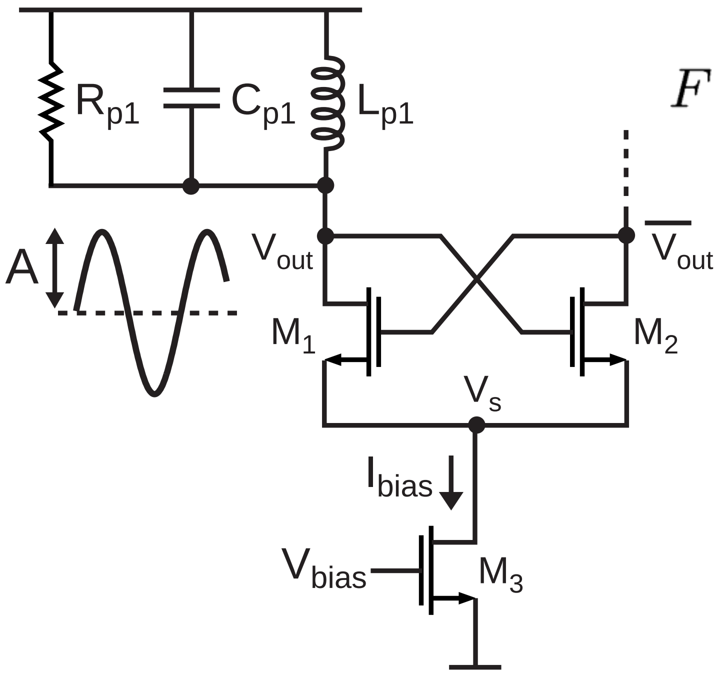
- **Rael et. al. have come up with a closed form expression for F(Δf) for the above topology**
- **In the region where phase noise falls at -20 dB/dec:**

$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \gamma \frac{4}{9} g_{do,M3} R_p \qquad (R_p = R_{p1} = R_{p2})$$

References to Rael Work

- Phase noise analysis
  - J.J. Rael and A.A. Abidi, “Physical Processes of Phase Noise in Differential LC Oscillators”, Custom Integrated Circuits Conference, 2000, pp 569-572
- Implementation
  - Emad Hegazi et. al., “A Filtering Technique to Lower LC Oscillator Phase Noise”, JSSC, Dec 2001, pp 1921-1930

Designing for Minimum Phase Noise



$$F(\Delta f) = \underbrace{1}_{(A)} + \underbrace{\frac{2\gamma I_{bias} R_p}{\pi A}}_{(B)} + \underbrace{\gamma \frac{4}{9} g_{do,M3} R_p}_{(C)}$$

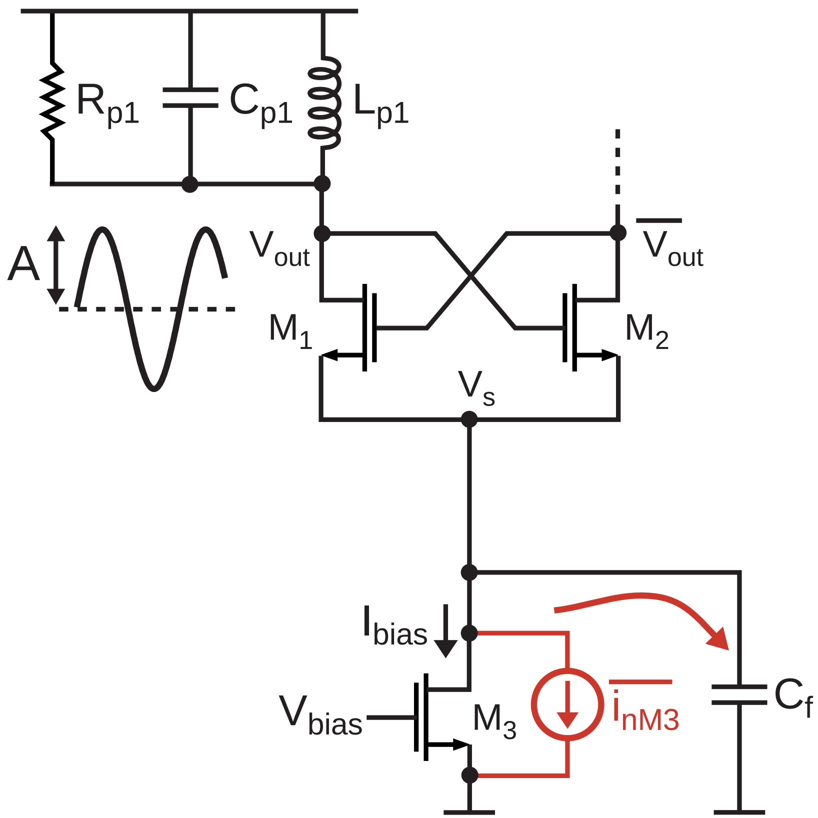
(A) Noise from tank resistance

(B) Noise from M_1 and M_2

(C) Noise from M_3

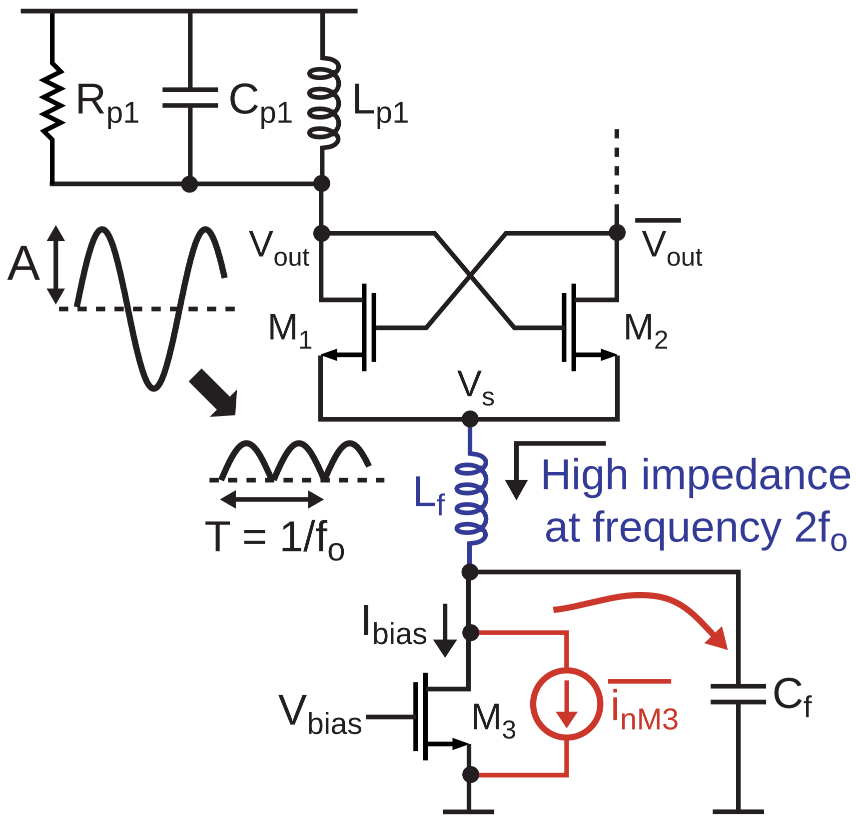
- **To achieve minimum phase noise, we'd like to minimize F(Δf)**
- **The above formulation provides insight of how to do this**
  - **Key observation: (C) is often quite significant**

Elimination of Component (C) in F(Δf)



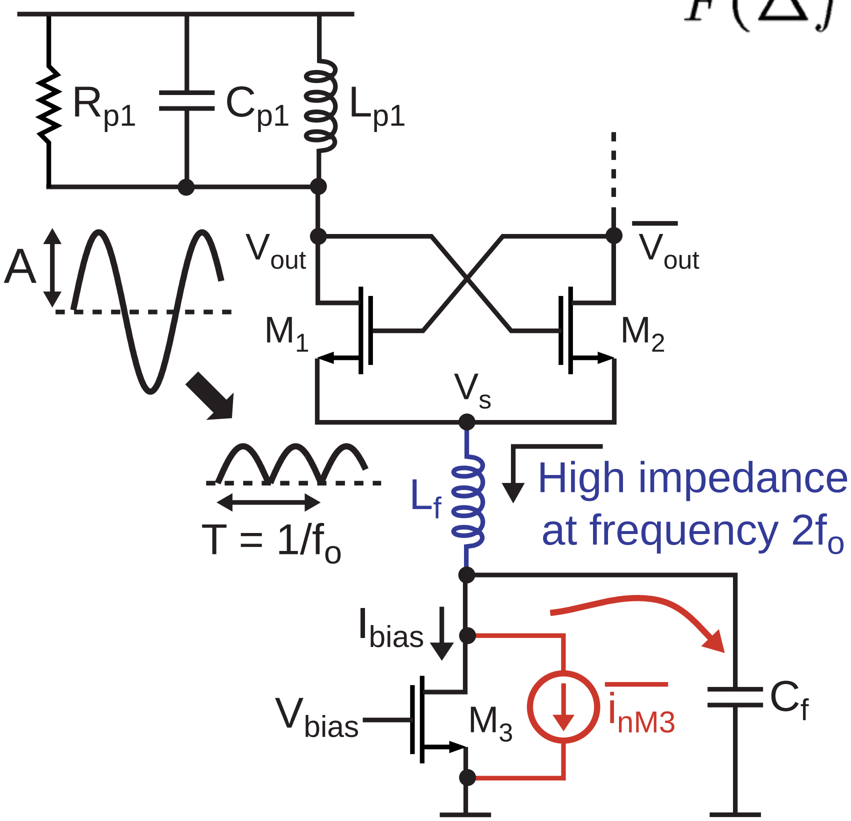
- Capacitor C_f shunts noise from M_3 away from tank
  - Component **(C)** is eliminated!
- Issue – impedance at node V_s is very low
  - Causes M_1 and M_2 to present a low impedance to tank during portions of the VCO cycle
    - Q of tank is degraded

Use Inductor to Increase Impedance at Node V_s



- Voltage at node V_s is a rectified version of oscillator output
  - Fundamental component is at twice the oscillation frequency
- Place inductor between V_s and current source
  - Choose value to resonate with C_f and parasitic source capacitance at frequency $2f_o$
- Impedance of tank not degraded by M_1 and M_2
  - Q preserved!

Designing for Minimum Phase Noise – Next Part



$$F(\Delta f) = \underbrace{1}_{(A)} + \underbrace{\frac{2\gamma I_{bias} R_p}{\pi A}}_{(B)} + \underbrace{\gamma \frac{4}{9} g_{do,M3} R_p}_{(C)}$$

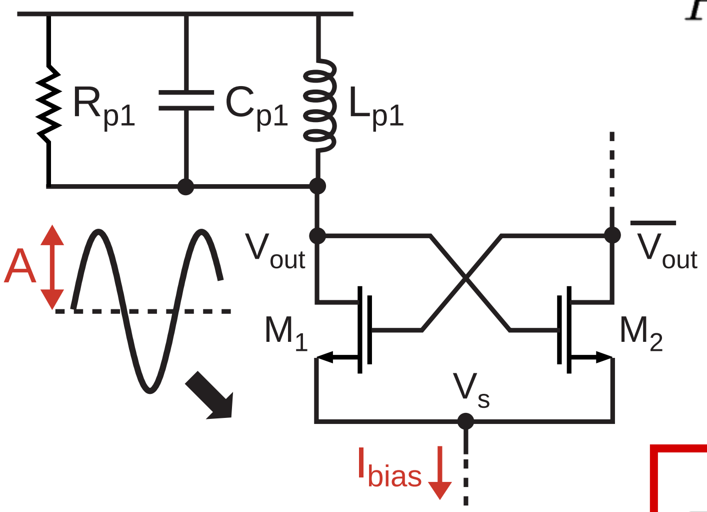
(A) Noise from tank resistance

(B) Noise from M_1 and M_2

(C) Noise from M_3

- **Let's now focus on component (B)**
  - **Depends on bias current and oscillation amplitude**

Minimization of Component (B) in F(Δf)



$$F(\Delta f) = 1 + \underbrace{\frac{2\gamma I_{bias} R_p}{\pi A}}_{(B)}$$

- **Recall from Lecture 11**

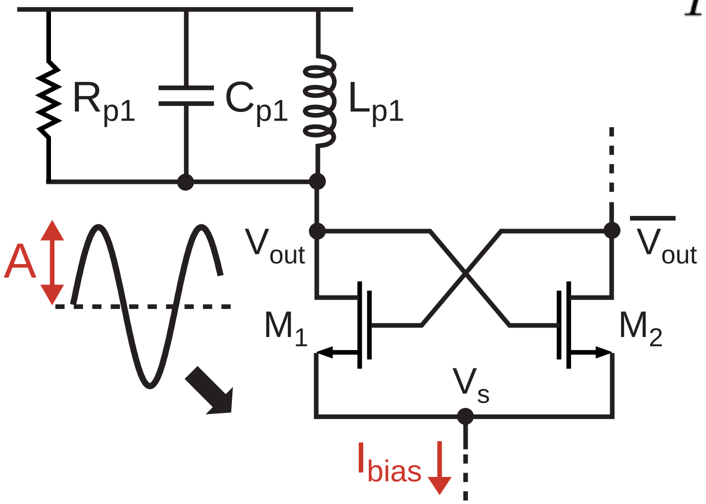
$$A = \frac{2}{\pi} I_{bias} R_p$$

$$\Rightarrow \boxed{F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi (2/\pi) I_{bias} R_p} = 1 + \gamma}$$

- **So, it would seem that I_{bias} has no effect!**
  - **Not true – want to maximize A (i.e. P_{sig}) to get best phase noise, as seen by:**

$$L(\Delta f) = 10 \log \left(\frac{2kTF(\Delta f)}{P_{sig}} \left(\frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \right)$$

Current-Limited Versus Voltage-Limited Regimes



$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A}$$

(B)

- **Oscillation amplitude, A, cannot be increased above supply imposed limits**
- **If I_{bias} is increased above the point that A saturates, then (B) increases**

- **Current-limited regime: amplitude given by** $A = \frac{2}{\pi} I_{bias} R_p$
- **Voltage-limited regime: amplitude saturated**

Best phase noise achieved at boundary between these regimes!

Final Comments

- **Hajimiri method useful as a numerical procedure to determine phase noise**
  - **Provides insights into 1/f noise upconversion and impact of noise current modulation**
- **Rael method useful for CMOS negative-resistance topology**
  - **Closed form solution of phase noise!**
  - **Provides a great deal of design insight**
- **Another numerical method**
  - **Spectre RF from Cadence now does a reasonable job of estimating phase noise for many oscillators**
    - Useful for verifying design ideas and calculations