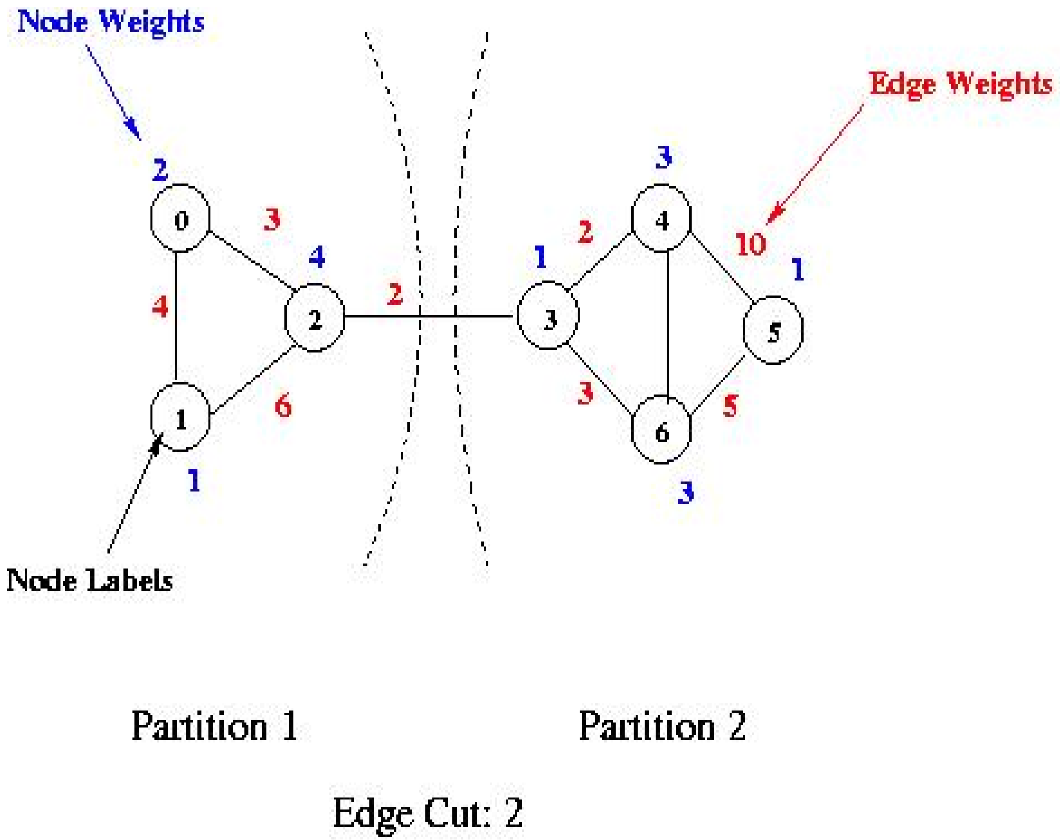
Parallelizing METIS

A Graph Partitioning Algorithm

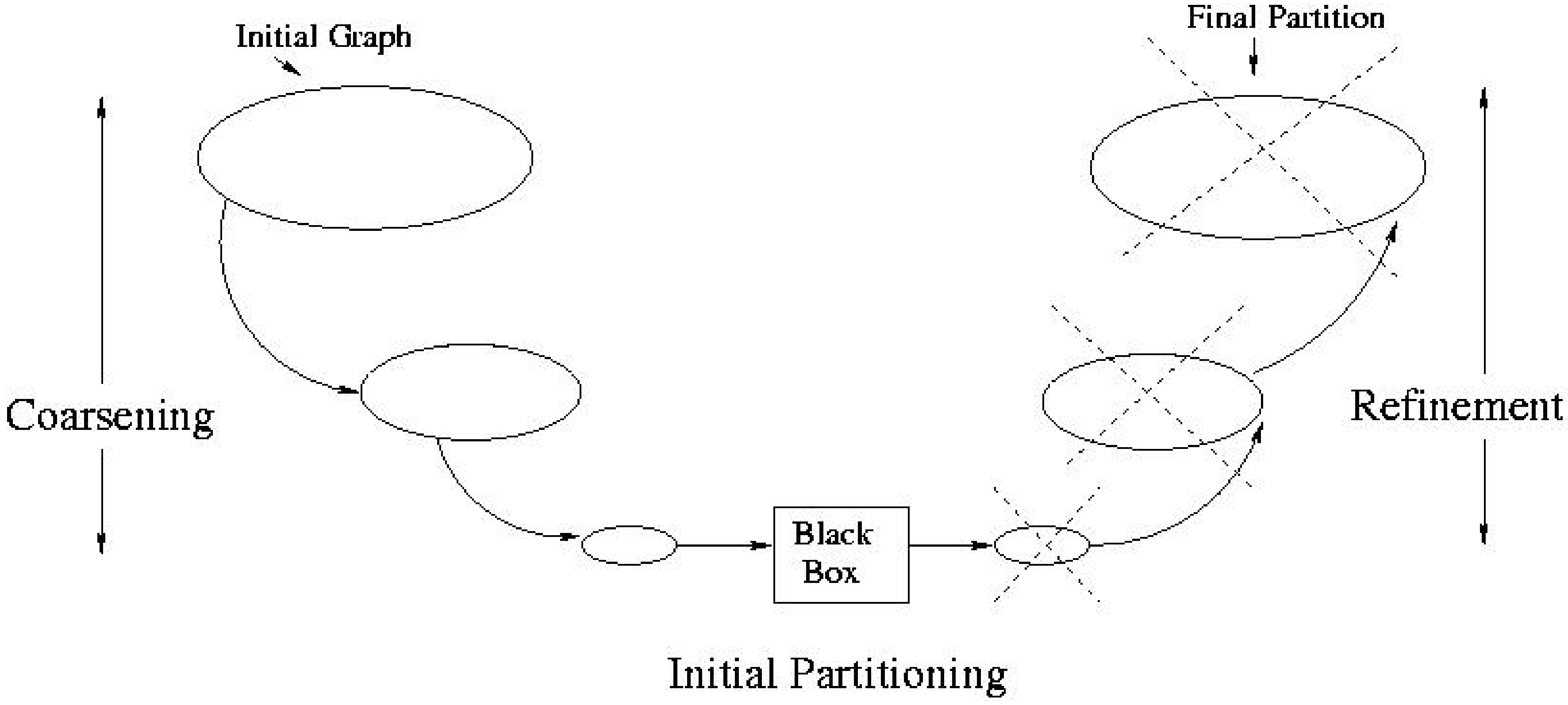
Zardosht Kasheff

Sample Graph

- Goal: Partition graph into n equally weighted subsets such that edge cut is minimized
- Edge-cut: Sum of weights of edges whose nodes lie in different partitions
- Partition weight: Sum of weight of nodes of a given partition.

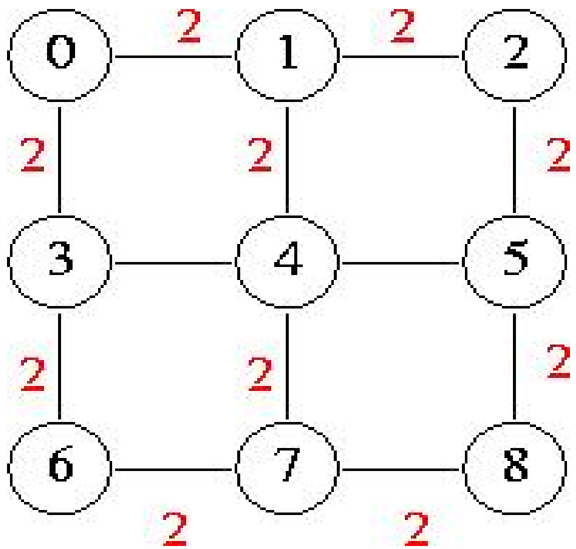


METIS Algorithm



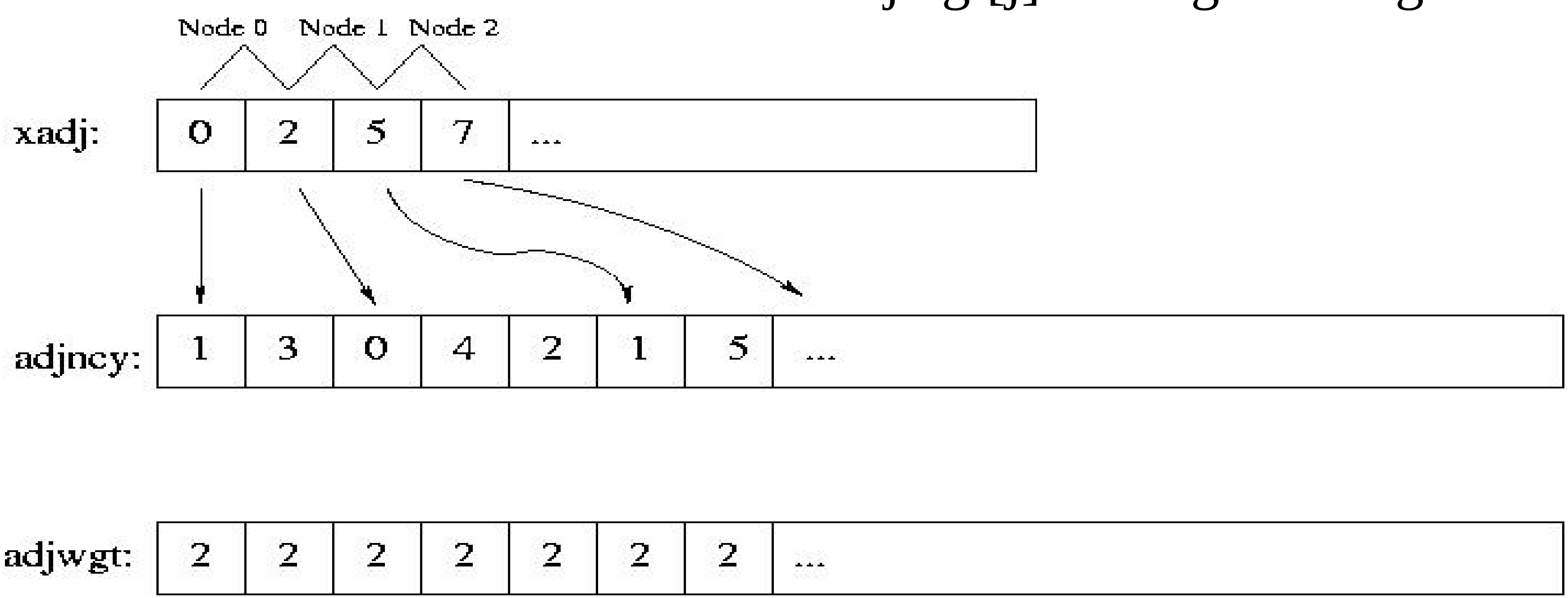
95% of runtime is spent on Coarsening and Refinement

Graph Representation

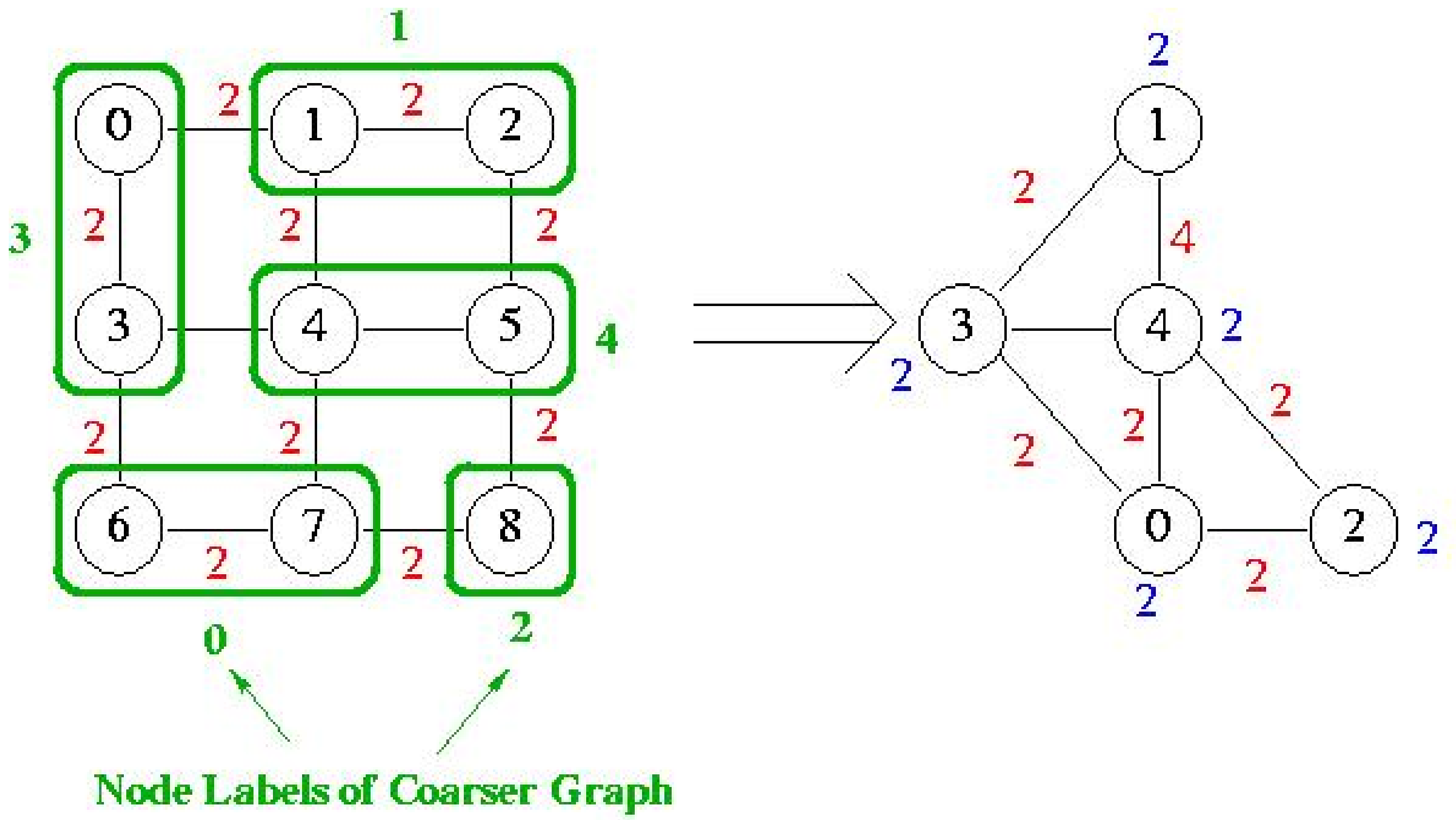


All data stored in arrays

- xadj holds pointers to adjncy and adjwgt that hold connected nodes and edge weights
- for j, such that xadj[i] <= j < xadj[i+1]: adjncy[j] is connected to i, adjwgt[j] is weight of edge connecting



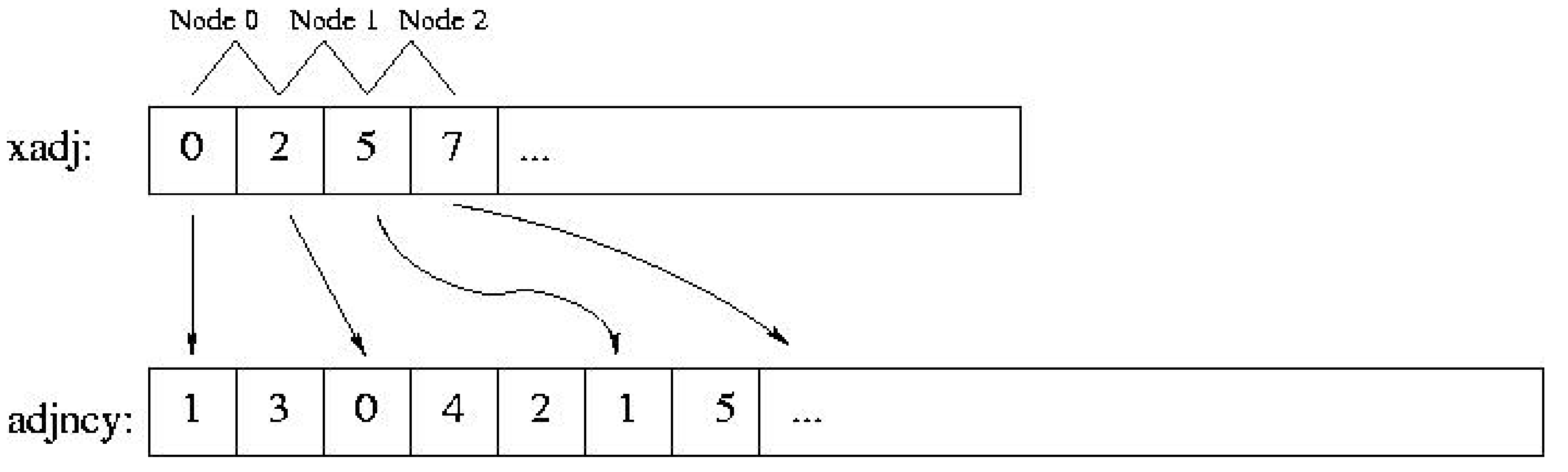
Coarsening Algorithm



Matching

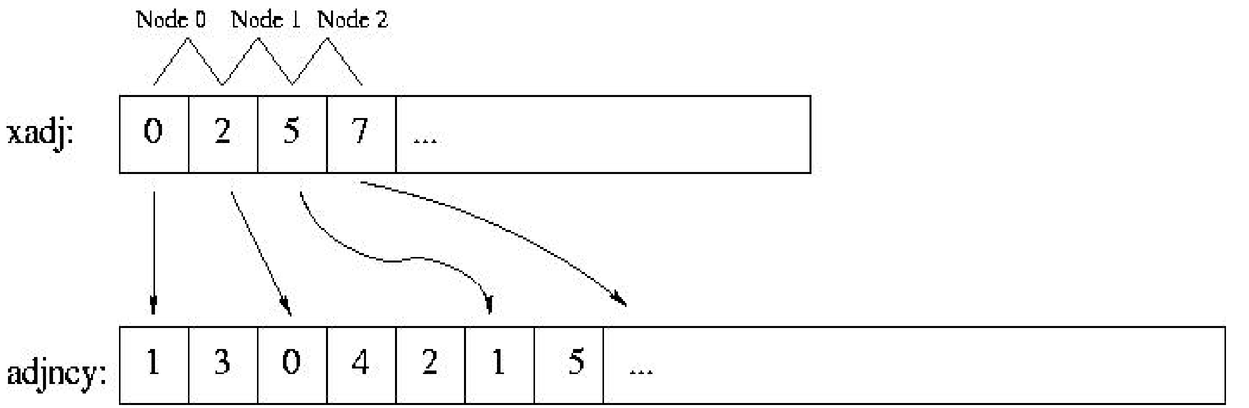
Writing Coarse Graph

Coarsening: Writing Coarse Graph Issue: Data Represention

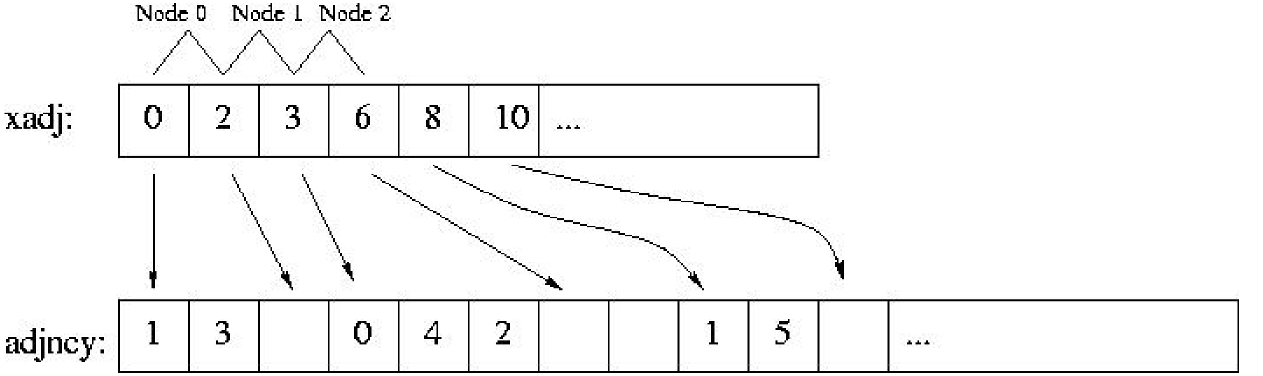


Coarsening: Writing Coarse Graph Issue: Data Represention

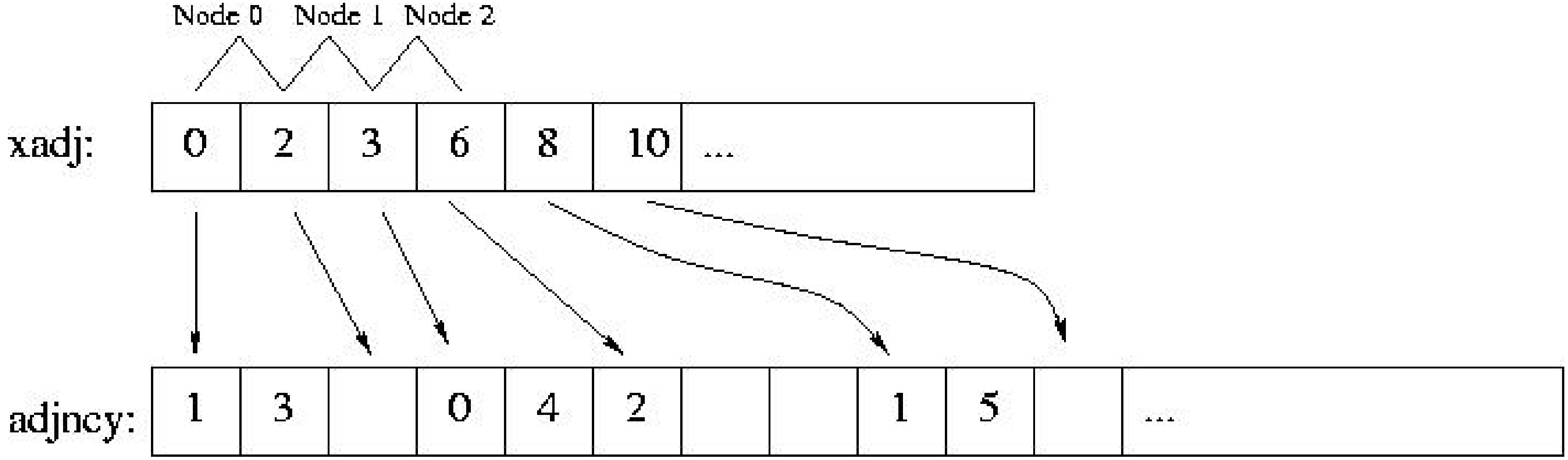
<u>Before</u>:
for j, such that
xadj[i] <= j < xadj[i+1]:
adjncy[j] connected to i.



<u>After</u>:
for j, such that
xadj[2i] <= j < xadj[2i+1]:
adjncy[j] connected to i.



Coarsening: Writing Coarse Graph Issue: Data Represention



- Now, only need *upper bound* on number of edges per new vertex
  - If match(*i*,*j*) map to *k*, then *k* has at most |edges(*i*)| + |edges(*j*)|
  - Runtime of preprocessing xadj only O(|*V*|).

Coarsening: Writing Coarse Graph Issue: Data writing

- Writing coarser graph involves writing massive amounts of data to memory
  - $T_1 = O(|E|)$
  - $T_\infty = O(\lg |E|)$
  - Despite parallelism, little speedup

Coarsening: Writing Coarse Graph Issue: Data writing

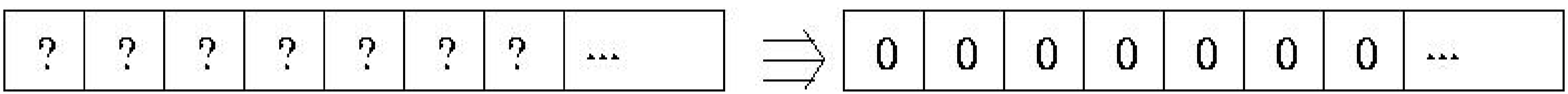
Example of filling in array:

```
Cilk void fill(int *array, int val, int len){
 if(len <= (1<<18)){
   memset(array, val, len*4);
  } else {
   /************RECURSE************/
  }
}
enum { N = 200000000 };
int main(int argc, char *argv[]){
  x = (int *) malloc(N*sizeof(int));
  mt_fill(context, x, 25, N);gettimeofday(&t2);print_tdiff(&t2, &t1);
  mt_fill(context, x, 25, N);gettimeofday(&t3);print_tdiff(&t3, &t2);
}
```

Coarsening: Writing Coarse Graph Issue: Data writing

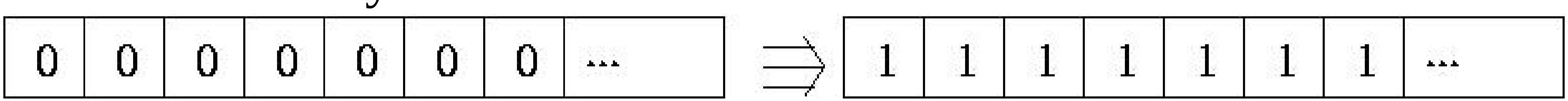
- Parallelism increases on second fill

After first malloc, we fill array of length 2*10^8 with 0's:



1 proc: 6.94s	
2 proc: 5.8s	speedup: 1.19
4 proc: 5.3s	speedup: 1.30
8 proc: 5.45s	speedup: 1.27

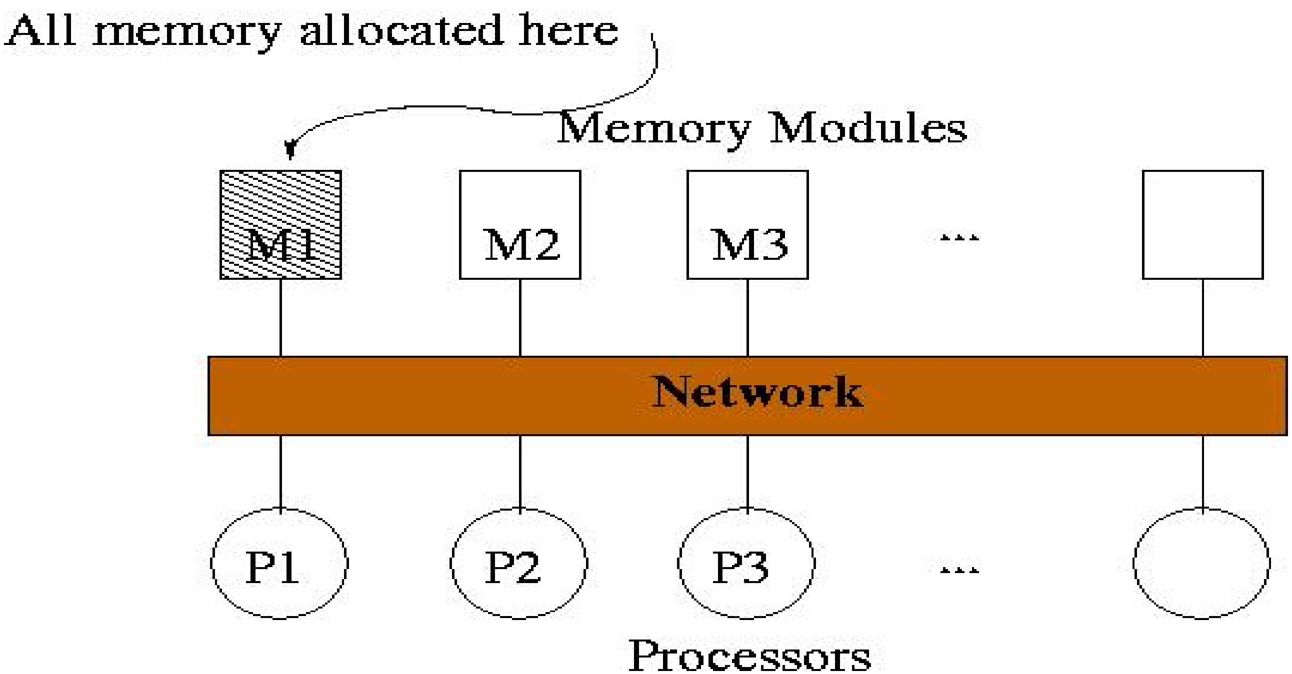
Then we fill array with 1's:



1 proc: 3.65s	
2 proc: 2.8s	speedup: 1.30
4 proc: 1.6s	speedup: 2.28
8 proc: 1.25s	speedup: 2.92

Coarsening: Writing Coarse Graph Issue: Data writing

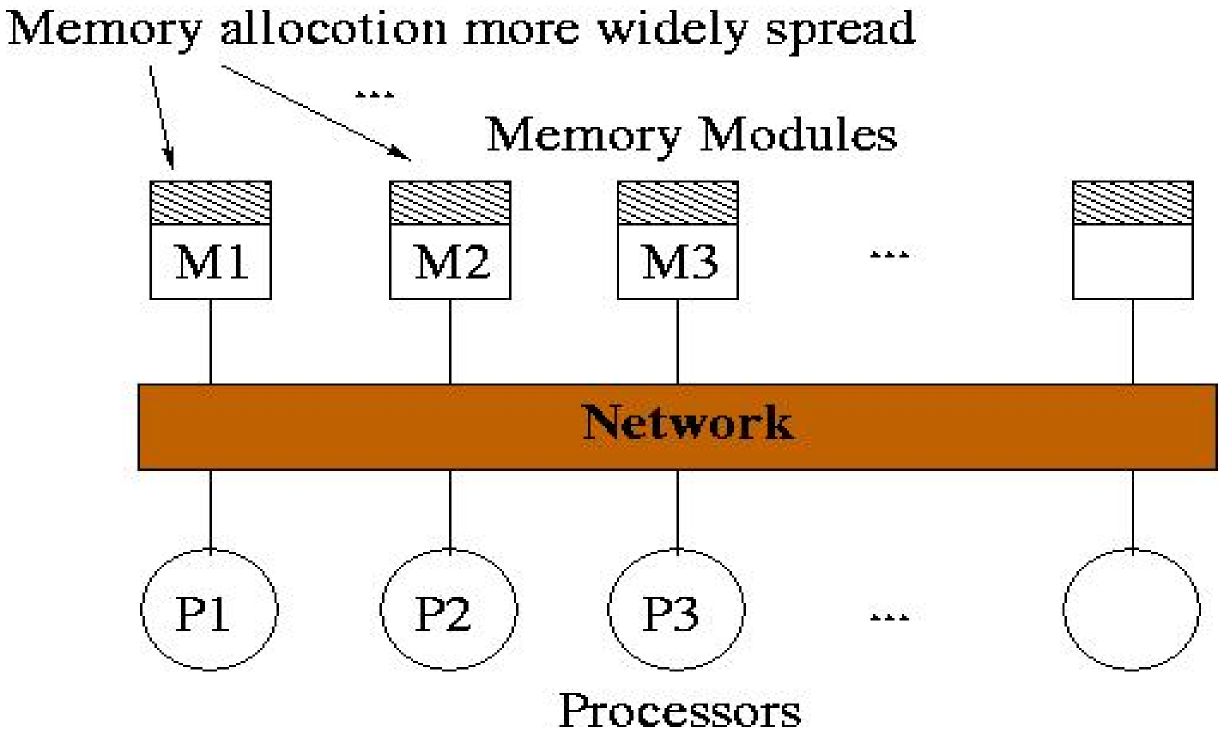
- Memory Allocation
  - Default policy is First Touch:
    - Process that first touches a page of memory causes that page to be allocated in node on which process runs



Result:
Memory Contention

Coarsening: Writing Coarse Graph Issue: Data writing

- Memory Allocation
  - Better policy is Round Robin:
    - Data is allocated in round robin fashion.



Result:
More total work but less memory contention.

Coarsening: Writing Coarse Graph Issue: Data writing

- Parallelism with round robin placement on ygg.

After first malloc, we fill array of length 2*10^8 with 0's:

?	?	?	?	?	?	?	...

⇒

0	0	0	0	0	0	0	...

1 proc: 6.94s		1 proc: 6.9s	
2 proc: 5.8s	speedup: 1.19	2 proc: 6.2s	speedup: 1.11
4 proc: 5.3s	speedup: 1.30	4 proc: 6.5s	speedup: 1.06
8 proc: 5.45s	speedup: 1.27	8 proc: 6.6s	speedup: 1.04

Then we fill array with 1's:

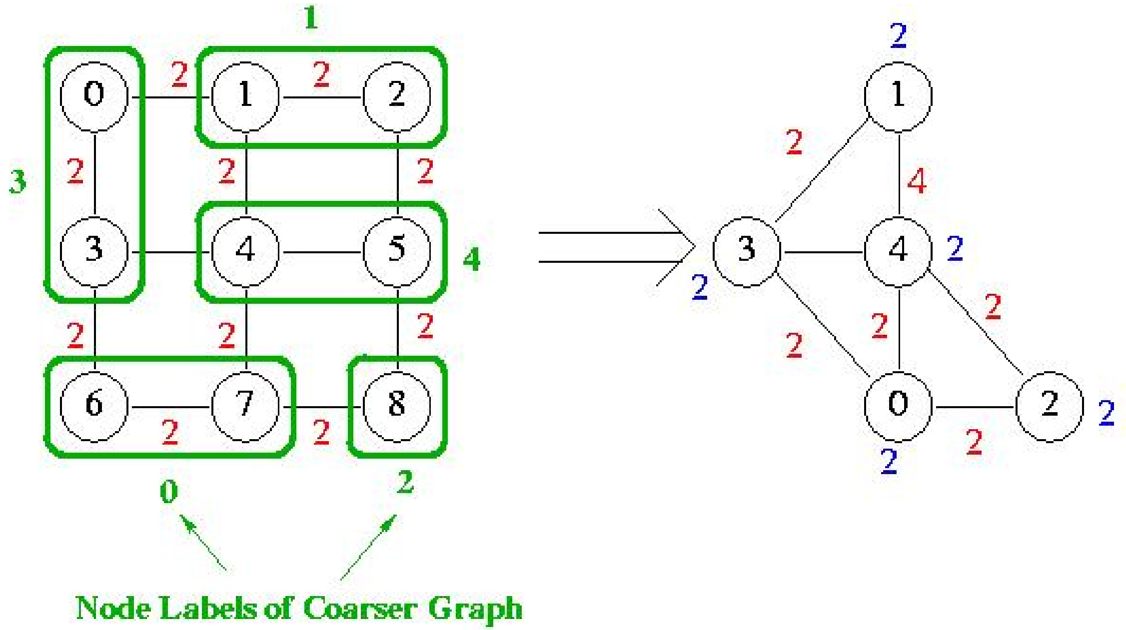
0	0	0	0	0	0	0	...

⇒

1	1	1	1	1	1	1	...

1 proc: 3.65s		1 proc: 4.0s	
2 proc: 2.8s	speedup: 1.3	2 proc: 2.6s	speedup: 1.54
4 proc: 1.6s	speedup: 2.28	4 proc: 1.3s	speedup: 3.08
8 proc: 1.25s	speedup: 2.92	8 proc: .79s	speedup: 5.06

Coarsening: Matching



Matching   Writing Coarse Graph

match:

3	2	1	0	5	4	7	6	8

cmap:

3	1	1	3	4	4	0	0	2

numedges:

0	5	5	2	5	7

Coarsening: Matching Phase: Finding matching

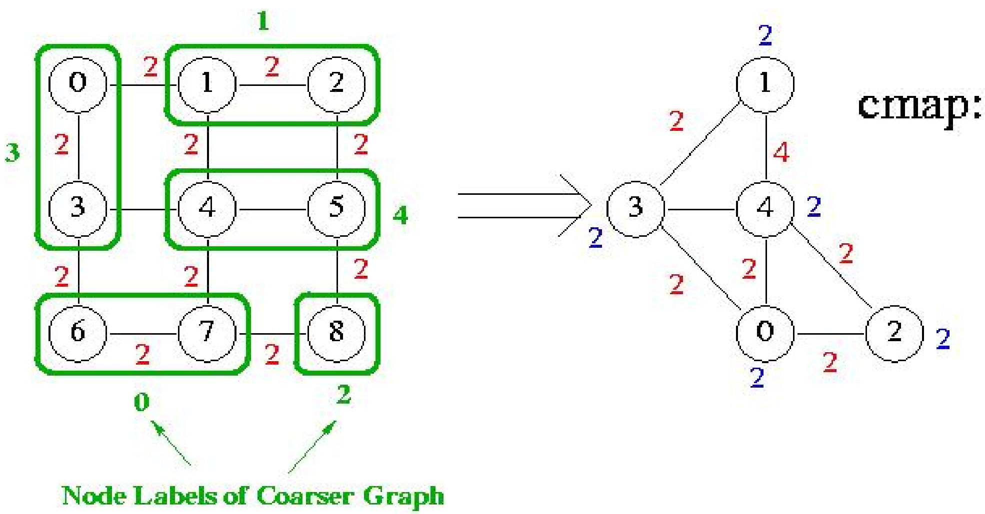
- Can use divide and conquer
  - For each vertex:

```
if(node u unmatched){
    find unmatched adjacent node v;
    match[u] = v;
    match[v] = u;
}
```

  - Issue: Determinacy races. What if nodes *i*,*j* both try to match *k*?
  - Solution: We do not care. Later check for all *u*, if match[match[*u*]] = *u*. If not, then set match[*u*] = *u*.

Coarsening: Matching Phase: Finding mapping

- Serial code assigns mapping in order matchings occur. So for:



Matching Writing Coarse Graph

cmap:

3	1	1	3	4	4	0	0	2

Matchings occurred in following order:

1) (6,7)
2) (1,2)
3) (8,8) /*although impossible in serial code, error caught in last minute*/
4) (0,3)
5) (4,5)

Coarsening: Matching Phase: Finding mapping

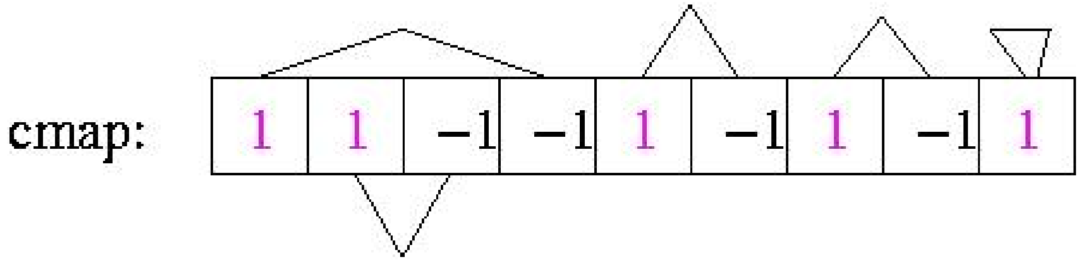
- Parallel code cannot assign mapping in such a manner without a central lock:
  - For each vertex:

```
if(node u unmatched) {
    find unmatched adjacent node v;
    LOCKVAR;
    match[u] = v;
    match[v] = u;
    cmap[u] = cmap[v] = num;
    num++;
    UNLOCK;
}
```

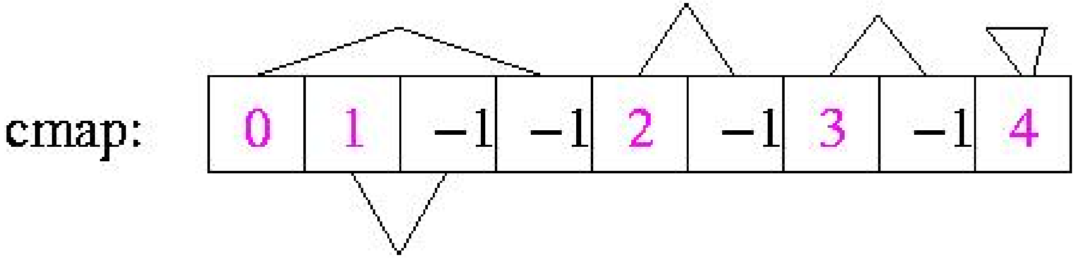
  - This causes bottleneck and limits parallelism.

Coarsening: Matching Phase: Finding mapping

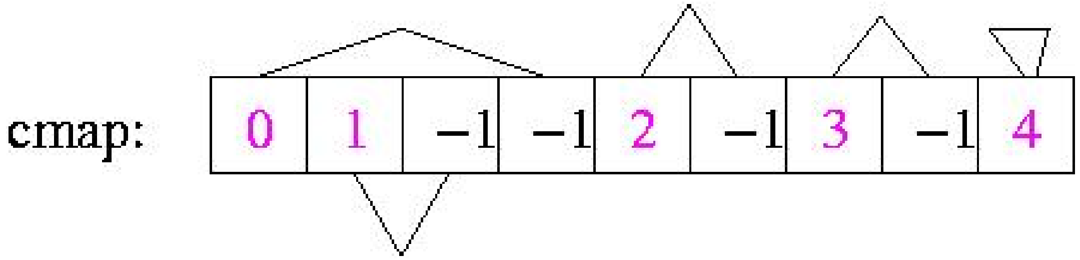
- Instead, can do variant on parallel-prefix
  - Initially, let cmap[*i*] = 1 if match[*i*] >= *i*, -1 otherwise:



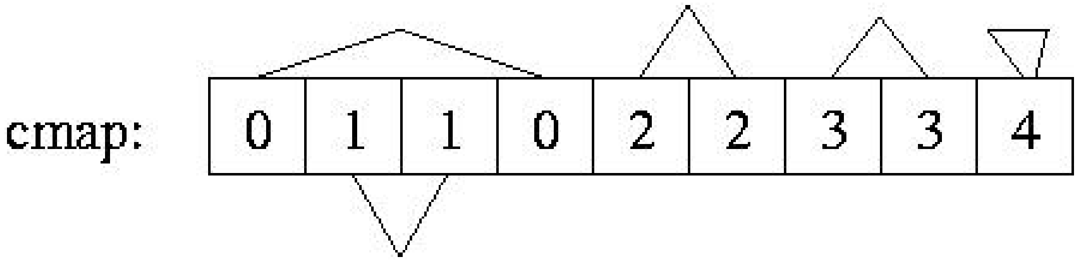
  - Run prefix on all elements not -1:



Coarsening: Matching Phase: Finding mapping



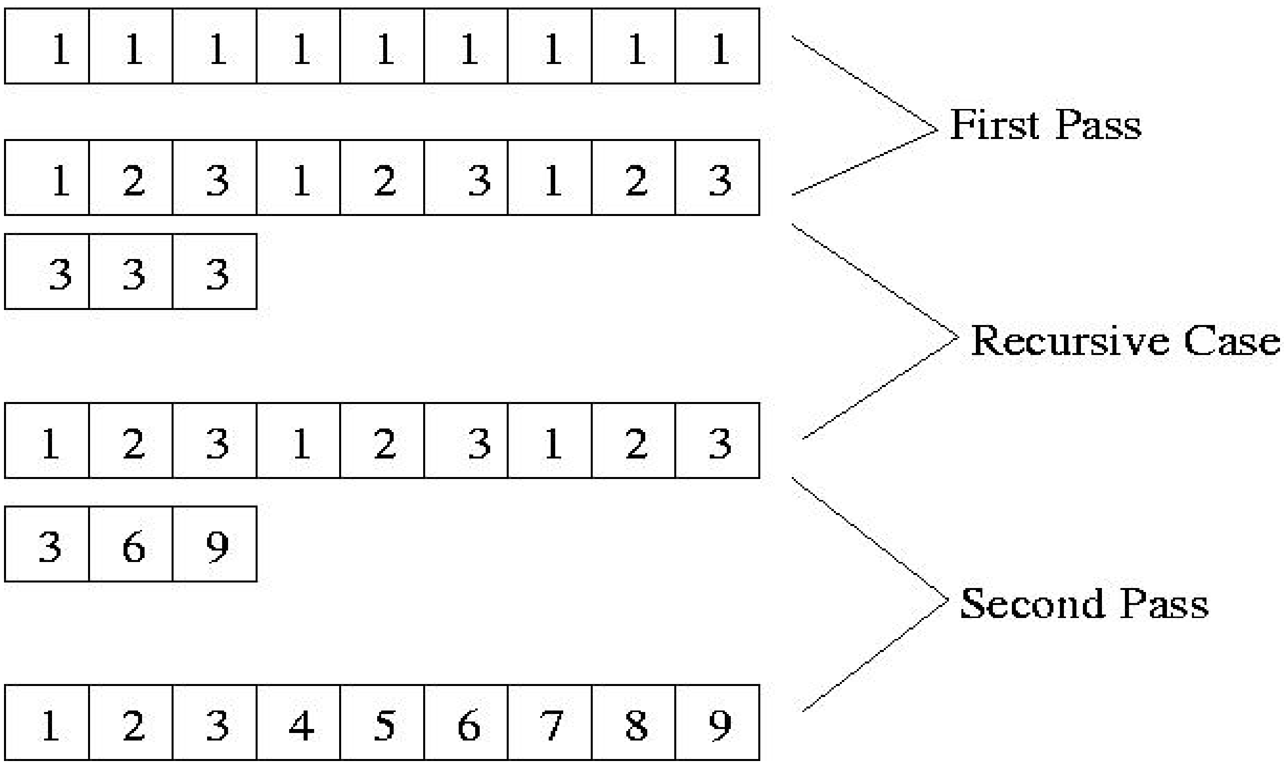
– Correct all elements that are -1:



– We do this last step after the parallel prefix to fill in values for cmap sequentially at all times. Combining the last step with parallel-prefix leads to false sharing.

Coarsening: Matching Phase: Parallel Prefix

- $T_1 = 2N$
- $T_{infinity\infty} = 2 \lg N$ where N is length of array.



Coarsening: Matching Phase: Mapping/Preprocessing xadj

- Can now describe mapping algorithm in stages:
  - First Pass:
    - For all *i*, if match[match[*i*]] != *i*, set match[*i*] = *i*
    - Do first pass of parallel prefix as described before
  - Second Pass:
    - Set cmap[*i*] if *i* <= match[*i*],
    - set numedges[cmap[*i*]] = edges[*i*] + edges[match[*i*]]
  - Third Pass:
    - Set cmap[*i*] if *i* > match[*i*]
- Variables in blue mark probable cache misses.

Coarsening: Preliminary Timing Results

On 1200x1200 grid, first level coarsening:

Serial:
Matching: .4s
Writing Graph: 1.2s

Parallel:

1proc:	2 proc	4 proc	8 proc
memsetting for matching: .17s			
matching: .42s	.23s	.16s	.11s
mapping: .50s	.31s	.17s	.16s
memsetting for writing: .44s			
coarsening: 1.2s	.71s	.44s	.24s

Round Robin Placement:

1proc:	2 proc	4 proc	8 proc
memsetting for matching: .20s			
matching: .51s	.27s	.16s	.09s
mapping: .64s	.35s	.20s	.13s
memsetting for writing: .52s			
coarsening: 1.42s	.75s	.39s	.20s