Massachusetts Institute of Technology Department of Materials Science and Engineering 77 Massachusetts Avenue, Cambridge MA 02139-4307

3.21 Kinetics of Materials—Spring 2006

March 15, 2006

Lecture 13: Atomic Models for Diffusivities.

## References

1. Balluffi, Allen, and Carter, Kinetics of Materials, Section 8.2.

2. Allen and Thomas, The Structure of Materials, Section 5.1.6, "Point Imperfections in Ionic Crystals."

## Key Concepts

- The relation  $D = \Gamma r^2 \mathbf{f}/6$  can be applied to various diffusion processes and the resulting expressions enable a more complete understanding of the dependence of D on temperature and other factors.
- The diffusivity of solute atoms by interstitial jumps in the b.c.c. structure is uncorrelated and the resulting expression for D takes the form  $D_I = (a^2/6)\nu \exp(S^m/K) \exp[-H^m/(kT)]$ .
- The vacancy diffusivity in the f.c.c. structure is uncorrelated and when the vacancies make nearestneighbor jumps the expression for D takes the form  $D_V = a^2 \nu \exp(\frac{S_V^m}{K}) \exp[-\frac{H_V^m}{kT}]$ .
- The self-diffusivity of a tracer atom in the f.c.c. structure by the vacancy mechanism involves jumps that can only occur when the tracer exchanges places with a neighboring vacancy. Thus, the vacancy concentration which at equilibrium is given by X<sub>V</sub> = exp(S<sup>f</sup><sub>V</sub>/K) exp[-H<sup>f</sup><sub>V</sub>/(kT)] must enter the desired expression for \*D. In addition, while the vacancy jumps will be uncorrelated, the tracer atom jumps will be correlated so the resulting expression for \*D takes the form \*D = fa<sup>2</sup>ν exp[(S<sup>m</sup><sub>V</sub> + S<sup>f</sup><sub>V</sub>)/K] exp[-(H<sup>m</sup><sub>V</sub> + H<sup>f</sup><sub>V</sub>)/(kT)]. Detailed analysis of the correlated jumping of the tracer atoms leads to the approximate expression f ≈ (z − 1)/(z + 1), where z is the coordination number for the crystal structure in which the tracers are jumping. A more complete analysis indicates that f ≈ 0.78 in f.c.c. crystals.
- The expressions for D written above all obey an Arrhenius law of the form  $D = D^{\circ} \exp[-E/(kT)]$ . Depending on the mechanism of diffusion, the activation energy E may have contributions from both defect migration and formation processes.
- The *isotope effect* refers to the small effect of mass of the diffusing species on diffusivity in tracer diffusion processes. For species of masses m₁ and m₂, \*D₁/\*D₂ = √m₂/m₁.
- Complexities of solute-atom diffusion by the vacancy mechanism in binary alloys can be appreciated with the *three-frequency model* for jumping, in which the vacancy jump rates may differ depending on whether the exchange is with the solute atom, a near-neighbor to the solute atom, or a more distant neighbor to the solute atom.

## Related Exercises in Kinetics of Materials

Review Exercises 8.12-8.15, pp. 199-201.