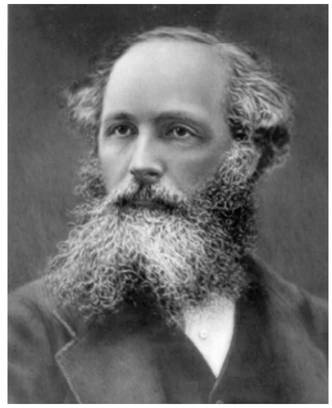
3.23 Electrical, Optical, and Magnetic Properties of Materials
Fall 2007

3.23 Fall 2007 – Lecture 16

MAXWELL AND ELECTROMAGNETISM



James Clerk Maxwell

Cavendish
Laboratory

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3.23 Electronic, Optical and Magnetic Properties of Materials - Nicola Marzari (MIT, Fall 2007)

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Last time

1. p-n junctions, built-in voltage, rectification
2. Bloch oscillations, conductivity in semiconductors
3. Electron transport at the nanoscale
4. Phonons, vibrational free energy, and the quasi-harmonic approximation
5. Electron-phonon interactions, and phonon-phonon decays

Study

- Fox, Optical Properties of Solids, Appendix A and Chap 1.
- Prof Fink lecture notes (to be posted)

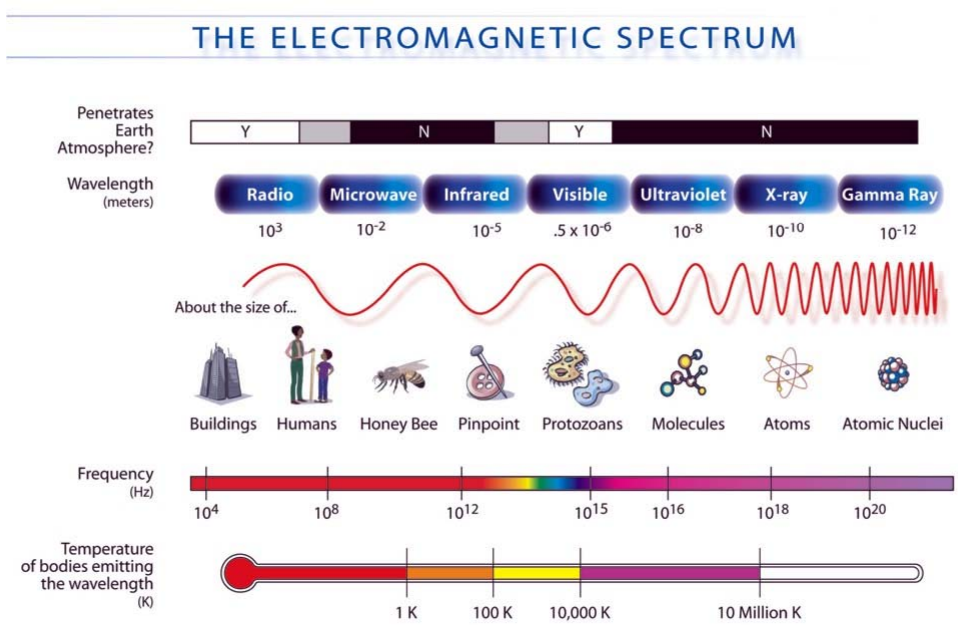
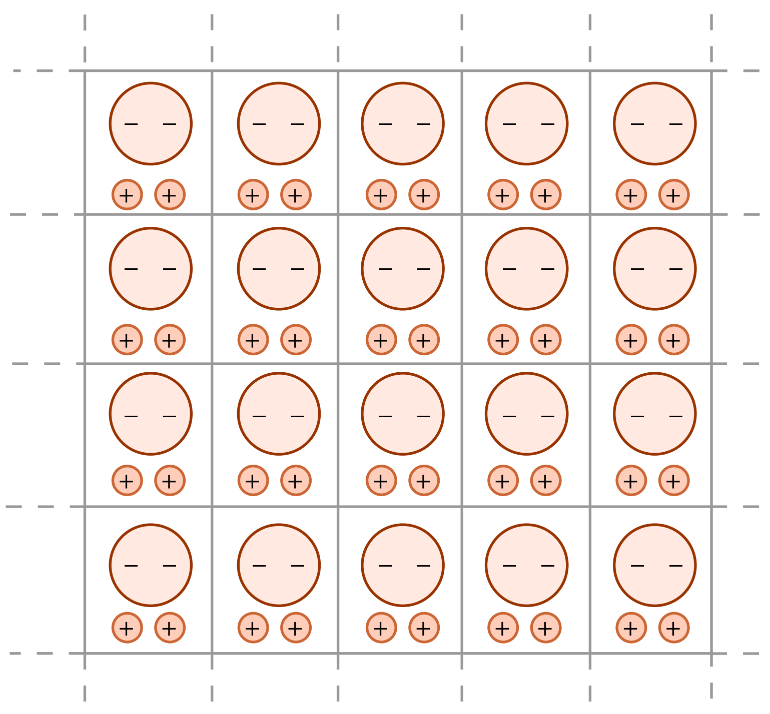


Image courtesy NASA.

Electric field, polarization, displacement



A complex crystal lattice can have a permanent intrinsic polarization P.

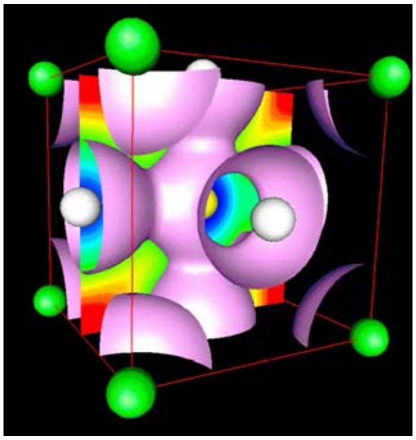
Figure by MIT OpenCourseWare.

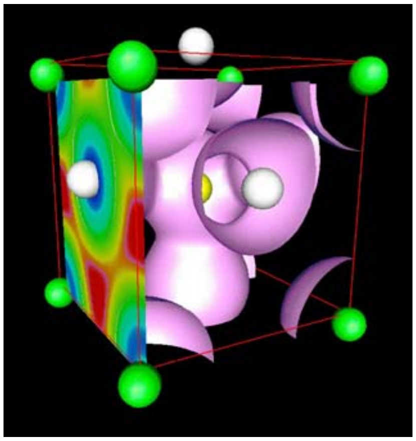
Lines & Glass, *Principles and Applications of Ferroelectrics and Related Materials (1977)*:

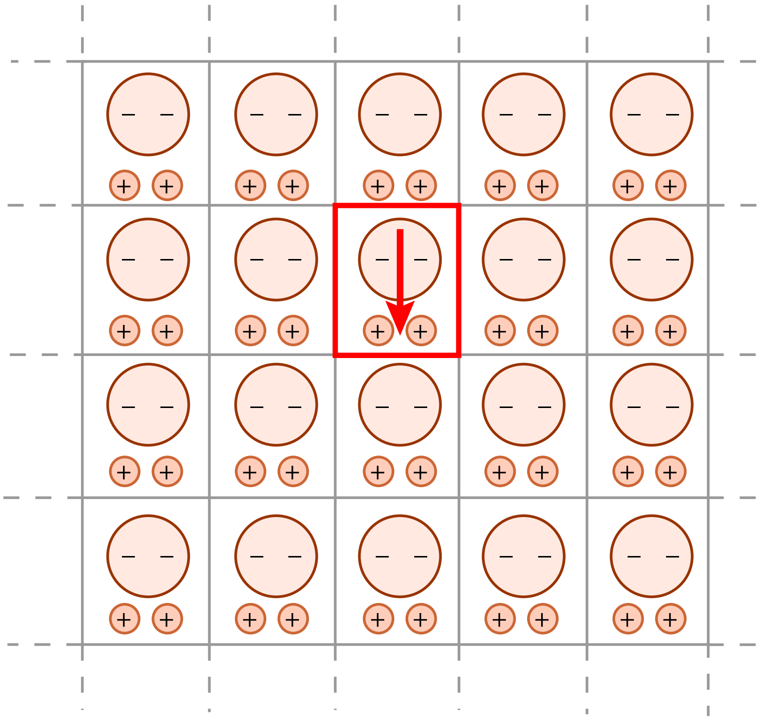
If and when good **electron-density maps** become available for ferroelectrics, expressing charge density $\rho(\mathbf{r})$ as a function of position vector **r** throughout the unit cell, more quantitative estimates of spontaneous polarization might be envisaged as

$$\mathbf{P}_s = \frac{1}{V}\int_V \mathbf{r}\rho(\mathbf{r})\, d\mathbf{r}. \qquad (6.1.19)$$

Polarization in lead titanate







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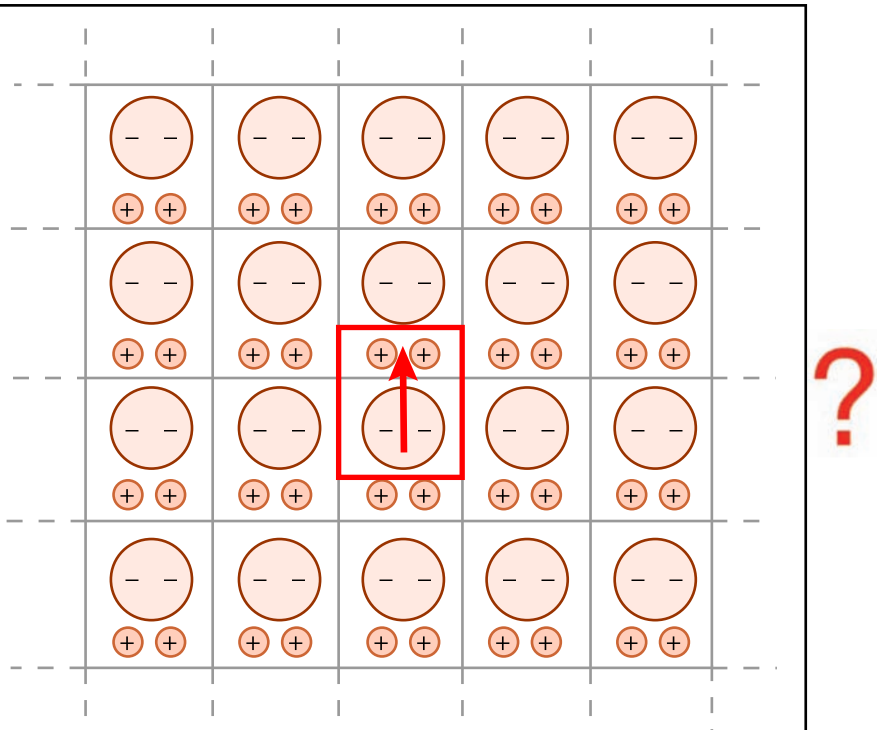
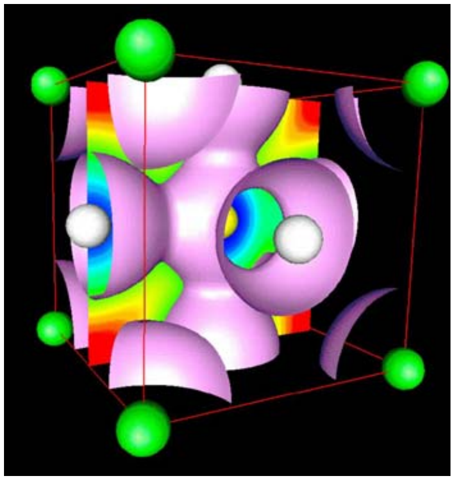
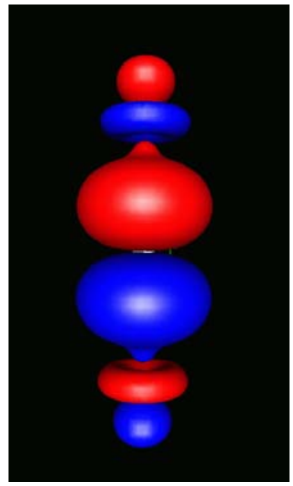


A complex crystal lattice can have a permanent intrinsic polarization P.

Figures by MIT OpenCourseWare.





Dielectric constant, susceptibility

Magnetic response

Maxwell equations

$$\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c} \vec{\ }$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{D} = 4\pi\rho$$

Vector potential and gauges

Vector potential and gauges

Summary

$$\vec{\nabla}\times\vec{E}+\frac{1}{c}\frac{\partial\vec{B}}{\partial t}=0$$

$$\vec{\nabla}\times\vec{H}-\frac{1}{c}\frac{\partial\vec{D}}{\partial t}=\frac{4\pi}{c}\vec{J}$$

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\cdot\vec{D}=4\pi\rho$$

E – electric field

H – magnetic field

D – electric displacement

B – magnetic displacement

$$\vec{\ } = \underbrace{\varepsilon}_{\text{dielectric tensor}} \vec{\ }$$

$$\vec{B} = \underbrace{\mu}_{\text{permeability tensor}} \vec{H}$$

$$\vec{D}=\varepsilon\vec{E}=\vec{E}+4\pi\vec{P}$$

$$\vec{B}=\mu\vec{H}=\vec{H}+4\pi\vec{M}$$

12 variables

8 scalar Maxwell equations

Electromagnetic waves

Electromagnetic waves

Summary

$$\vec{\nabla}\times\vec{E}+\frac{1}{c}\frac{\partial\vec{B}}{\partial t}=0\xrightarrow{\cdot\frac{1}{\mu}}\frac{1}{\mu}\vec{\nabla}\times\vec{E}+\frac{1}{c}\frac{\partial\vec{H}}{\partial t}=0\xrightarrow{\cdot\vec{\nabla}\times}\vec{\nabla}\times\left(\frac{1}{\mu}\vec{\nabla}\times\vec{E}\right)+\frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\times\vec{H}=0$$

$$\vec{\nabla}\times\vec{H}-\frac{1}{c}\frac{\partial\vec{D}}{\partial t}=0\xrightarrow{\cdot\frac{\partial}{\partial t}}\frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\times\vec{H}=\frac{\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\vec{\nabla}\times\left(\frac{1}{\mu}\vec{\nabla}\times\vec{E}\right)+\frac{\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}=0$$

$$\vec{\nabla}^2\vec{E}-\frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}=0$$

$$\vec{\nabla}^2\vec{H}-\frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{H}}{\partial t^2}=0$$

$$\vec{\nabla}^2\vec{E}=\frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\vec{E}(x,y,z,t)=\vec{E}_0e^{i\omega t-\vec{k}\cdot\vec{r}}$$

$$\frac{c}{\sqrt{\mu\varepsilon}}\left|\vec{k}\right|=\omega$$

Refractive index

Phase velocity

Wave packets