3.23 Electrical, Optical, and Magnetic Properties of Materials
Fall 2007

3.23 Fall 2007 – Lecture 14

INHOMOGENEOUS SEMICONDUCTORS

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Russell Ohl **Shockley, Bardeen, and Brattain**

William Shockley

Electronic Bands in Sodium Chloride

(advisor John C. Slater, MIT, 1936)

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3.23 Electronic, Optical and Magnetic Properties of Materials - Nicola Marzari (MIT, Fall 2007)

ELECTRONIC BANDS IN SODIUM CHLORIDE

BY

WILLIAM SHOCKLEY

B.Sc., California Institute of Technology
1932

Submitted in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

from the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
1936

Signature of Author..

Department of Physics, May 14, 1936.

Signature of Professor
in Charge of Research..

Signature of Chairman of Department
Committee on Graduate Students..............................

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Last time

1. Band structure of direct- and indirect-gap semiconductors, in excruciating detail
2. Carriers in thermal equilibrium, density of available states
3. Law of mass action
4. Consequences for intrinsic semiconductors, extrinsic semiconductors
5. Impurity levels, hydrogen model of donors, acceptor states
6. Temperature dependence of majority carriers: intrinsic range, extrinsic/saturation range, freeze out.

Study

- Early part of Chap 29 (Inhomogeneous semiconductors) ,Ashcroft-Mermin (to be posted, together with Chap 28, really to be posted, s'il vous plaît)

Density of available states

$$g_c(\varepsilon) = \sqrt{2(\varepsilon - \varepsilon_c)}\frac{m_c^{3/2}}{\pi^2\hbar^3}$$

$$\Uparrow$$

$$\underbrace{\int_{\varepsilon_c}^{\infty} d\varepsilon\, g_c(\varepsilon) e^{-(\varepsilon-\varepsilon_c)/k_B T}}$$

$$N_c(T) = \frac{1}{4}\left(\frac{2m_c k_B T}{\pi\hbar^2}\right)^{3/2} = 2.5\left(\frac{m_c}{m}\right)^{3/2}\left(\frac{T}{300K}\right)^{3/2} 10^{19}/cm^3$$

Law of Mass Action

$$n_c p_v = N_c P_v e^{-E_g/k_B T}$$

Intrinsic case

$$n_i = \sqrt{n_c p_v} = \sqrt{N_c P_v}\, e^{-E_g/2k_B T}$$

$$n_c(T) = N_c(T)\, e^{-(\varepsilon_c - \mu)/k_B T}$$

$$p_v(T) = P_v(T)\, e^{-(\mu - \varepsilon_v)/k_B T}$$

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Please see: Fig. 11 in Sze, S. M. "Physics of Semiconductor Devices." Chapter 1 in *Physics and Properties of Semiconductors - A Resume.* New York, NY: John Wiley & Sons, 1981.

$$\mu_i = \varepsilon_v + \frac{1}{2}E_g + \frac{3}{4}k_B T \ln\left(\frac{m_v}{m_c}\right)$$

Extrinsic case

$$n_c(T) - p_v(T) = \Delta n \qquad n_i$$

$$\begin{matrix} n_c \\ p_v \end{matrix} = \frac{1}{2}\left[(\Delta n)^2 + 4n_i^2\right]^{1/2} \pm \frac{1}{2}\Delta n$$

$$\Downarrow$$

$$\left[n_c^2 + p_v^2 - 2n_c p_v + 4 n_c p_v\right]^{1/2}$$

$$\frac{1}{2}(n_c + p_v) \pm \frac{1}{2}(n_c - p_v)$$

Extrinsic case

$$n_c = \left[e^{\beta(\mu-\mu_i)}\right] n_i \qquad p_v = \left[e^{-\beta(\mu-\mu_i)}\right] n_i$$

$$\frac{n_c - p_v}{n_i} = \frac{\Delta n}{n_i} = 2 \sinh \beta(\mu - \mu_i)$$

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Temperature dependence of majority carriers

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Impurity types, levels

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Please see: Fig. 13 in Sze, S. M. "Physics of Semiconductor Devices." Chapter 1 in *Physics and Properties of Semiconductors - A Resume.* New York, NY: John Wiley & Sons, 1981.

Population of impurity levels (donor)

$$\langle n \rangle = \frac{\sum_j N_j e^{-(E_j - \mu N_j)\beta}}{\sum_j e^{-\beta(E_j - \mu N_j)}} = \frac{2e^{-\beta(\varepsilon_d - \mu)}}{1 + 2e^{-\beta(\varepsilon_d - \mu)}}$$

E_j N_j

$N_j = 0$ OR

$N_j = 1$ (UP) (DOWN)

$N_j = 2$ TOO EXPENSIVE (COULOMBIC REPULSION)

$$\langle n \rangle = \frac{1}{1 + \frac{1}{2}e^{\beta(\varepsilon_d - \mu)}}$$

$$n_d = N_d \langle n \rangle$$

Population of impurity levels (acceptor)

$$p_a = N_a \langle p \rangle = \frac{N_a}{1 + \frac{1}{2} e^{\beta(\mu - \varepsilon_a)}}$$

Conductivity in semiconductors

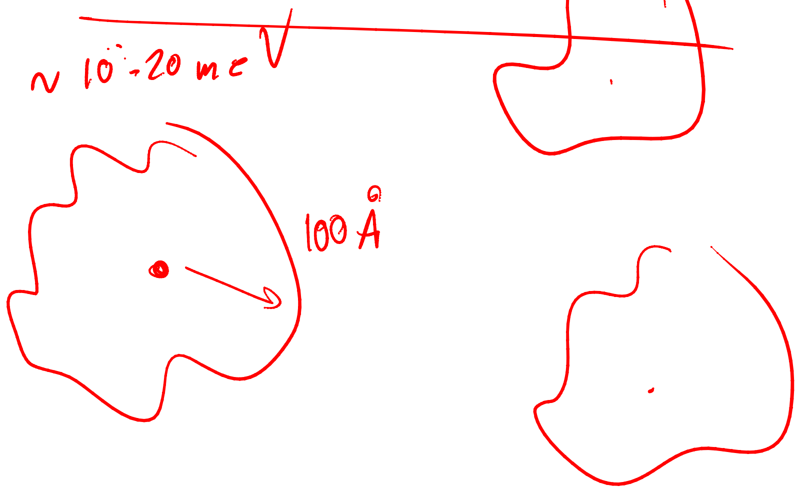
$$\sigma = n_e e \frac{e\tau_e}{m_e} + n_h e \frac{e\tau_h}{m_h}$$

$$\mu_e = \frac{e\tau_e}{m_e}$$

$$\mu_h = \frac{e\tau_h}{m_e}$$

Please see: Table 3 in Kittel, Charles. "Introduction to Solid State Physics." Chapter 8 in *Semiconductor Crystals.* New York, NY: John Wiley & Sons, 2004.

Impurity band conduction



Equilibrium carrier densities of impure semiconductors

N_d $\quad N_a \quad$ $N_d > N_a$

$$\underbrace{n_c + n_d}_{\text{\# electrons in conduction + \# electrons in donor state}} = \underbrace{N_d - N_a}_{T=0} + \underbrace{p_v + p_a}_{\text{\# of empty levels in valence and acceptor state}}$$

Equilibrium carrier densities of impure semiconductors

NON-DEG LIMITS

$\varepsilon_d - \mu \gg k_B T \quad n_d \ll N_d$

$\mu - \varepsilon_a \gg k_B T \quad p_a \ll N_a$

$$\Delta n = n_c - p_v = N_d - N_a$$

$$\begin{matrix} n_c \\ p_v \end{matrix} = \frac{1}{2}\left[(N_d - N_a)^2 + 4n_i^2\right]^{\frac{1}{2}} \pm \frac{1}{2}\left[N_d - N_a\right]$$

$$\frac{N_d - N_a}{n_i} = 2\sinh\beta(\mu - \mu_i)$$

Semiconductor carrier engineering

- Adding impurities to determine carrier <u>type</u>
  - n_i^2 for Si: ~10^{20}cm^{-3}
  - Add 10^{16}cm^{-3} (~1ppm) phosphorous (donors) to Si: n_c~N_d
  - n_c~10^{16}cm^{-3}, p_v~10^4 (n_i^2/N_d)
- Adding impurities to change carrier density
  - 1 part in 10^6 impurity in a crystal (~10^{22}cm^{-3} atom density)
  - 10^{22}/ 10^6 =10^{16} dopant atoms per cm^{-3}
  - conductivity is proportional to the # of carriers leading to 6 orders of magnitude change in conductivity!

Impurities at the ppm level drastically change the conductivity (5-6 orders of magnitude)

Simplified expressions

LOW IMPURITY CONCENTRATIONS

$$\left.\begin{matrix} n_c \\ p_v \end{matrix}\right\} = n_i \pm \frac{1}{2}(N_d - N_a)$$

$$n_c \simeq N_d - N_a$$

$$p_v \simeq \frac{n_i^2}{N_d - N_a}$$

$$N_d \geq N_a$$

$$n_c \simeq \frac{n_i^2}{N_a - N_d}$$

$$p_v = N_a - N_d$$

$$N_a \geq N_d$$

Abrupt junction

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Please see: Fig. 29.1 in Ashcroft, Neil W., and Mermin, N. David.
Solid State Physics. Belmont, CA: Brooks/Cole, 1976.

The p-n junction (diode)

<u>p-type material at equilibrium</u>

$p \sim N_A$

$n \sim n_i^2/N$

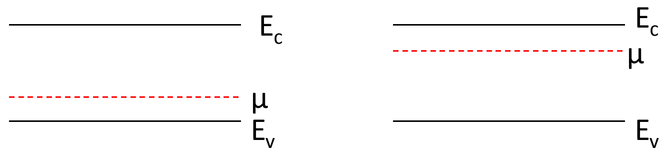
$$\mu_p = \mu_i - k_b T \ln\left(\frac{N_A}{n_i}\right)$$

<u>n-type material at equilibrium</u>

$n \sim N_D$

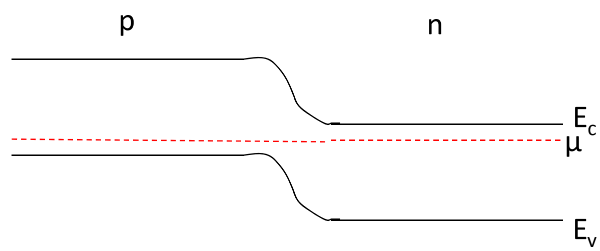
$p \sim n_i^2/N$

$$\mu_n = \mu_i + k_b T \ln\left(\frac{N_D}{n_i}\right)$$

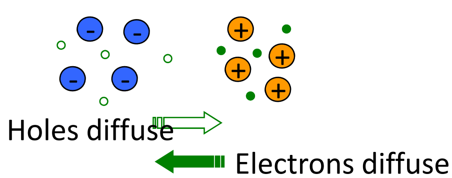


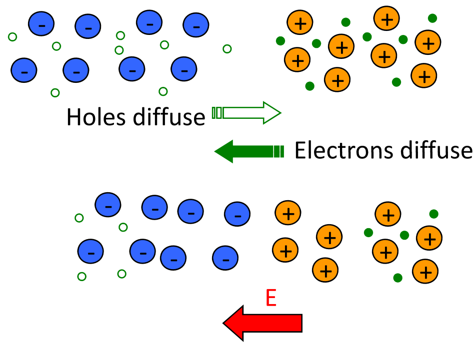
What happens when you join these together?

Joining p and n



Carriers flow under driving force of diffusion until μ is horizontal





An electric field forms due to the deviation from charge neutrality

Therefore, a steady-state balance is achieved where diffusive flux of the carriers is balanced by the drift flux

Chemical potential

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Please see: Fig. 29.2 in Ashcroft, Neil W., and Mermin, N. David.
Solid State Physics. Belmont, CA: Brooks/Cole, 1976.

Carrier concentration in a p-n junction

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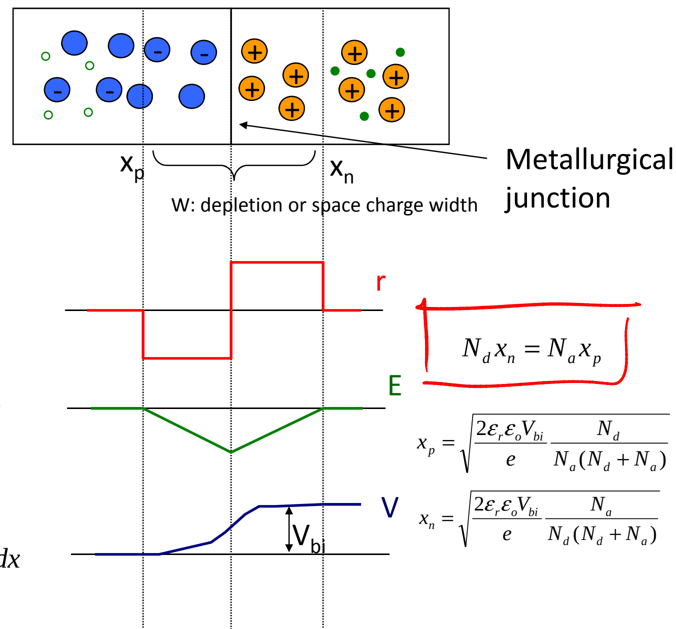


$$E = \int \frac{\rho(x)}{\varepsilon} dx$$

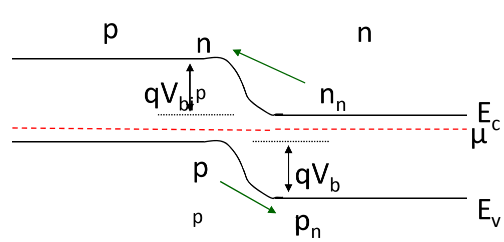
$$V = -\int E(x) dx$$

$$N_d x_n = N_a x_p$$

$$x_p = \sqrt{\frac{2\varepsilon_r \varepsilon_o V_{bi}}{e} \frac{N_d}{N_a (N_d + N_a)}}$$

$$x_n = \sqrt{\frac{2\varepsilon_r \varepsilon_o V_{bi}}{e} \frac{N_a}{N_d (N_d + N_a)}}$$

What is the built-in voltage V_{bi}?



$$qV_{bi} = \mu_n - \mu_p$$

$$\mu_p = \mu_i - k_b T \ln\left(\frac{N_a}{n_i}\right) \qquad \mu_n = \mu_i + k_b T \ln\left(\frac{N_D}{n_i}\right)$$

$$\therefore V_{bi} = \frac{k_b T}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

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Please see: Fig. 29.4 in Ashcroft, Neil W., and Mermin, N. David.
Solid State Physics. Belmont, CA: Brooks/Cole, 1976.

Operation under bias

Rectification

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Please see: Fig. 29.5 in Ashcroft, Neil W., and Mermin, N. David.
Solid State Physics. Belmont, CA: Brooks/Cole, 1976.