PROBLEMS Chapter 3.

3.1) Use the Lorentz force to describe how the angular frequency ω changes for a circular electron orbit (e.g. a classical Bohr orbit) in the xy plane when a magnetic field is applied along the z axis.

3.2) Derive the expressions for total energy E_n and radius of circular electron orbits of radius r_n in Bohr's model. Evaluate the constants E_1 and r_1 in SI units.

3.3) Calculate the classical angular momentum and the magnetic moment for a uniform shell of charge -|e| and radius $r = e^2 / (4\pi\varepsilon_0 mc^2)$ rotating with angular velocity ω. What is the gyromagnetic ratio for this particle? What is the surface velocity? (This problem was considered by Max Abraham in 1903, more than 20 years before Uhlenbeck and Goudsmit hypothesized that electrons spin with a gyromagnetic ratio twice that for orbital motion).

3.4) Compare essential characteristics of Lenz's Law (macroscopic) and diamagnetism. How are they similar and how different?

3.5) Calculate the dimension at which diamagnetism crosses over to paramagnetism for a metal with $\sigma = ne^2\tau/m = (ne^2/m)\ \lambda/\langle v\rangle = 10^7\ (\Omega\text{m})^{-1}$. ($\tau$ = relaxation time, λ = mean free path, $\langle v\rangle$ = mean drift velocity of change carriers.

3.6) a) Evaluate the ratio of integrals in the classical expression in the notes for $\langle cos\theta \rangle$

$$\frac{\int_0^{\pi} \exp\left(\frac{\mu_m H}{kT}\cos\theta\right)\cos\theta\sin\theta d\theta}{\int_0^{\pi} \exp\left(\frac{\mu_m H}{kT}\cos\theta\right)\sin\theta d\theta}$$

b) Show that $L(s) = \coth(x)-1/s$ goes to $x/3$ in the limit that x approaches zero. Discuss.

c) Show that $L(x) \to 1$ for $x = \infty$; discuss.

3.7) Show that the units of $\chi = N_v \mu_0 \mu_m^2 / kT$ and $\chi = N_v \mu_0 e^2 / 6m \sum r^2$ are $m^3 \times N_v$ so that if N_v is number of atoms per unit volume, χ is dimensionless. However, if N is Avogadro's number. (in which case χ is the molar susceptibility χ_{mol}) it must be multiplied by $10^6 \chi \text{m}^3/\text{m}^3$ to compare with $4\pi\chi_{\text{mol}} = 4\pi N \mu_m^2 / kT$, or $4\pi N e^2 / 6mc^2 \sum r^2$ calculated in cgs units.

3.8) Calculate the paramagnetic susceptibility of diatomic oxygen at room temperature and compare it with the experimental, room temperature (cgs) value $\chi_{\text{mol}} = 3.4 \times 10^{-3}$.

3.9) Calculate the diamagnetic susceptibility of atomic He assuming $r^2 \approx a_0^{\,2}$ and compare with the room temperature cgs value $\chi_{\text{mol}} = -1.88 \times 10^{-6}$.

3.10) Derive a classical expression for the diamagnetic susceptibility of an electron in circular orbit by considering the change in its angular momentum due to the electric field induced as a B field is slowly turned on normal to the orbit plane in time dt.

3.11) Analyze the condition for which the paramagnetic and diamagnetic susceptibilities are equal and opposite for a classical Bohr atom with orbital but not spin magnetic moment. Discuss.

3.12) Calculate the orbital magnetic moment of an electron in a circular Bohr orbit. Use $\omega = E/\hbar = 1.36\text{eV/h}$ and then use $\omega = v/r = (2E/m)^{1/2}/r_0$ and compare the results. Which one is correct? Why?

3.13) A very thin film (t < mean free path) may exhibit diamagnetism in an external field perpendicular to the film plane. Explain how application of the field in the plane of the film could change the sign of this effect.

3.14) Write the electronic configuration (e.g. $3d^4$), the spectroscopic notation ($^{2s+1}L_J$ e.g. $^3D_{5/2}$) and effective magneton number $n_{eff} = g\,[J(J+1)]^{1/2}$ where $g = 1 + [J(J+1) + S(S+1) - L(L+1)]/2J(J+1)$ for Cr^{3+}, Fe^{3+} and Co^{2+}.

3.15) a) Show that $B_j(x)$ reduces to the Langevin function $L(x)$, with $x = \mu_m B/kT$, in the limit J approaches infinity.

b) Show that $B_{1/2}(x) = tanh\,(x)$

c) Show that for x <<1, $B_j(x)$ becomes

$$B_J(x) \xrightarrow{x<<1} \frac{J(J+1)}{3J^2}\,x$$

and thus in this limit

$$\chi = \mu_o \frac{N_v g^2 \mu_B^2 J(J+1)}{3kT}$$

d) Show that as x approaches infinity, $B_j(x)$ approaches 1, i.e. $M = N_v g\, \mu_B\, m_J$. Describe the physical significance of each case.

3.16) The Landé g factor is used to account for the fact that $\boldsymbol{\mu_J} = \boldsymbol{\mu_L} + \boldsymbol{\mu_S}$ is not collinear with $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ because $\mu_L = \mu_B\, m_l = (eh/2m) m_l$ whereas $\mu_s = 2\, \mu_B m_s$. Derive the expression for g in terms of the quantum numbers l, s and j. Make use of the facts that because ***L*** and ***S*** precess around ***J*****,** then $\boldsymbol{\mu_L}$, $\boldsymbol{\mu_S}$ and $\boldsymbol{\mu_J}$ also precess around ***J*** and it is the projection of $\boldsymbol{\mu_J}$ on ***J*** that is measured.

3.17) What type(s) of magnetism would you expect to find and why in a) NaCl b) $MnSO_4 \cdot 4H_2O$ c) Fe_3O_4 d) H_2O (or Ne) e) metallic Cu?