3.60 Symmetry, Structure and Tensor Properties of Materials

Problem Set 13

1. There are three monoclinic point groups: 2, m and 2/m. Symmetry theory says that an inversion center added to 2 results in symmetry 2/m. Similarly, an inversion center added to symmetry m results in 2/m. We have shown that the symmetry of inversion imposes no constraints on a second-order property tensor, and consequently that the constraints for second-order tensors will be alike for a monoclinic crystal of any symmetry.

In class we chose 2 as the representative symmetry for study. Show that symmetry m requires exactly the same constraints as symmetry 2. SUGGESTION: Rather than actually derive the restrictions element by element (although this would be an acceptable response), you might consider the nature of the direction cosines involved in these transformations of axes.

2. When an electric field, E, is applied to a dielectric material, a polarization, P, (defined as dipole moment per unit volume) results. The polarization is related to the applied field by the dielectric susceptibility tensor, k_{ij}

$$P_i = k_{ij} E_j$$

Let's again examine aragonite, an orthorhombic form of $CaCO_3$, point group 2/m 2/m 2/m, with lattice constants a = 4.94, b = 7.94, c = 5.72 Å. Measured relative to the crystal axes, the susceptibility is given by

$$k_{ij} = \varepsilon_o \begin{bmatrix} 8.8 & 0 & 0 \\ 0 & 6.7 & 0 \\ 0 & 0 & 5.6 \end{bmatrix}$$

where ε_o, the permittivity of space, is $8.85 \; 10^{-12}$ coulombs/volt-meter.

Suppose an electric field of 10^5 volts/meter is applied along the [111] direction of a crystal aragonite.

(a) What is the magnitude of the polarization developed?

(b) What is the direction of P? (i.e., determine the direction cosines of P)

(c) What is the angle between P and E?

(d) What is the value of k in the [111] direction?

3. Show that a symmetric second-order tensor ($a_{ij} = a_{ji}$) remains symmetric after transformation to a new set of reference axes x_i' whose orientation is specified by a general direction cosine matrix $[c_{ij}]$.