3.60 Symmetry, Structure and Tensor Properties of Materials

Problem Set 18

1. The engineering moduli of crystals may all be expressed in terms of the elements of the stiffness constant matrix c_{ij} (or, alternatively, the compliances s_{ij}).

Show, for example, that for a cubic crystal the bulk modulus

$$B = -V(dP/dV)$$

is given by

$$B = \frac{c_{11} + 2c_{12}}{3}.$$

2. In June 1971 R. L. Pober submitted a thesis entitled "The Effect of Hydrostatic Pressure on Grain Boundary Self-Diffusion in Lead" for the Sc.D. in Ceramics. His experiments consisted of measuring grain boundary diffusion coefficients in bicrystals of lead (face-centered cubic structure) subjected to a hydrostatic pressure through immersion in an oil bath.

It is known that even cubic crystals are anisotropic in their elastic behavior. Dr. Pober was therefore concerned about the possibility of stress being induced at the grain boundary as a result of differring deformation in the two single crystals which composed the separate halves of the bicrystal.

Let the <u>linear compressibility</u> of a crystal, β, be defined as the relative decrease in the length of a line when a crystal is subjected to unit hydrostatic pressure. In general, β varies with direction. For a hydrostatic pressure p, the stress tensor is given by

$$[\sigma ij] = \begin{matrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{matrix}$$

The stretch of a line in a direction of a crystal is given by the "value" of ε in that direction, that is,

$$\varepsilon = \varepsilon_{ij}\ell_i\ell_j.$$

Since $$\varepsilon_{ij} = s_{ijk}\sigma_k$$

then $$\varepsilon_{ij} = -s_{ijkk}p$$

and $$\varepsilon = -p\ (s_{ijkk})\ell_i\ell_j$$

from which $$\beta = s_{ijkk}\ell_i\ell_j$$

(a) Derive an expression for the linear compressibility of a cubic crystal as a function of direction.

(b) Show that this value is independent of crystal direction, and thus that a sphere of cubic crystal will remain a sphere under compression and, consequently, that the bicrystals will not fall apart.