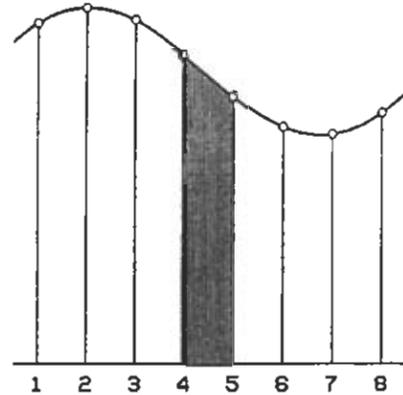


- 10 Somewhat in the spirit of Newton-Cotes, suppose we wished to evaluate accurately for any 7th-degree polynomial  $f(x)$  its integral between  $x = 4$  and 5 from just the eight samplings at the places shown.

What rational numbers  $w_k = p/q$  should we then use for the weights in

$$\int_{x=4}^5 f(x) dx = \sum_{k=1}^8 w_k f(k) \quad ?$$



- 11 Don't let the sum  $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \dots$  drive you bananas!

Instead try several tail-integration and/or Aitken strategies — or, best of all, employ Euler-Maclaurin — to conquer it smartly to at least 9 decimals.

- 12 Ditto for  $P = \prod_{n=3}^{\infty} \cos(\pi/n) = \cos 60^\circ \cdot \cos 45^\circ \cdot \cos 36^\circ \cdot \cos 30^\circ \dots$