

2.004: MODELING, DYNAMICS, & CONTROL II

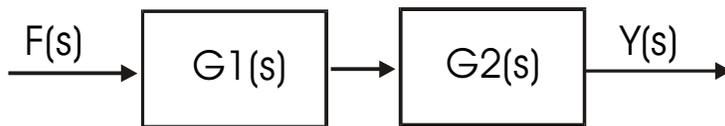
Fall Term 2002

PLEASE ALSO NOTE THAT ALL PRELAB EXERCISE ARE DUE AT THE START (WITHIN 10 MINUTE) OF THE LAB SESSION, NO LATE WORK IS ACCEPTED.

Pre-Lab Exercise for Experiment 7

Reducing the following block diagram to a single transfer function.

(a) Cascade form:

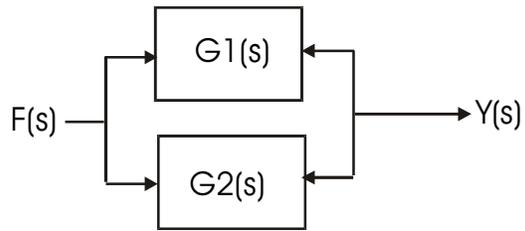


$$Y(s) = G2(s) G1(s) F(s)$$

$$T(s) = Y(s)/F(s)$$

$$T(s) = G1(s) G2(s)$$

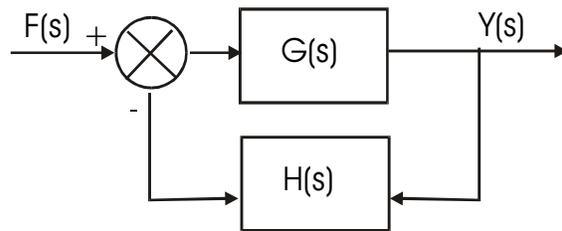
(b) Parallel form:



$$Y(s) = G1(s) F(s) + G2(s) F(s)$$

$$T(s) = G1(s) + G2(s)$$

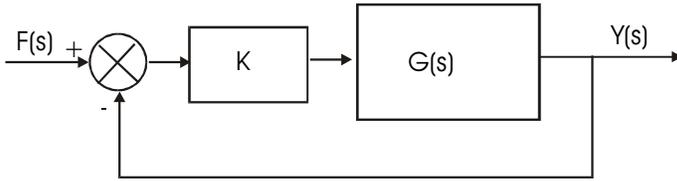
(c) Feedback form:



$$Y(s) = G(s) (F(s) - H(s) Y(s))$$

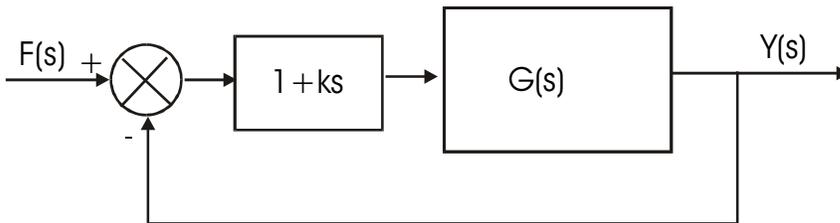
$$T(s) = \frac{G(s)}{1 + H(s)G(s)}$$

(d) Proportional cascade feedback form



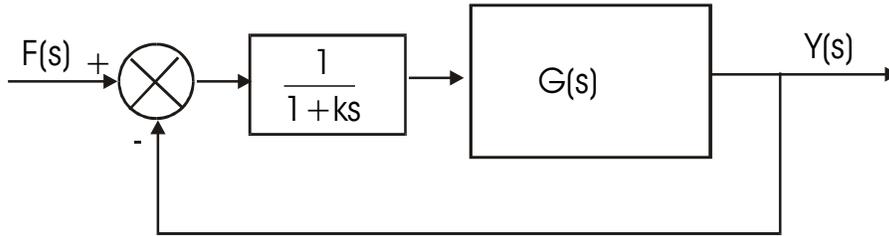
$$T(s) = \frac{K * G(s)}{1 + K * G(s)}$$

(e) Differential cascade feedback form:



$$T(s) = \frac{(1 + ks)G(s)}{1 + G(s) + ksG(s)}$$

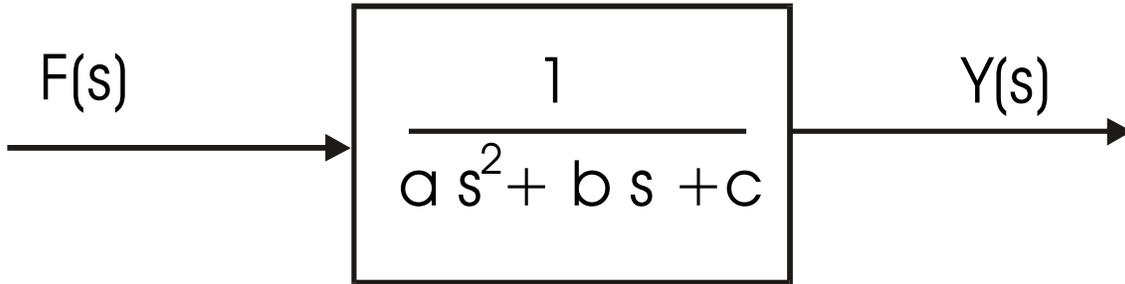
(f) Integral cascade feedback form:



$$T(s) = \frac{G(s)}{1 + ks + G(s)}$$

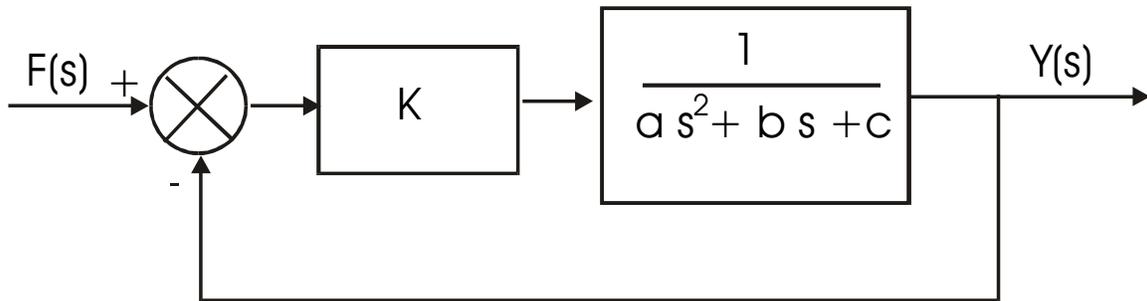
(b) Show that the transfer function can be expressed as:

$$\frac{Y(s)}{F(s)} = \frac{1}{as^2 + bs + c}$$



um, yeah.

(c) Consider the following feedback scheme with a proportional cascade compensator. Reduce the block diagram into a single block. Express the poles and zeros of the transfer function as a function of a, b, c, & K. Estimate the steady state error.



$$T(s) = \frac{K}{as^2 + bs + c + K}$$

no zeros

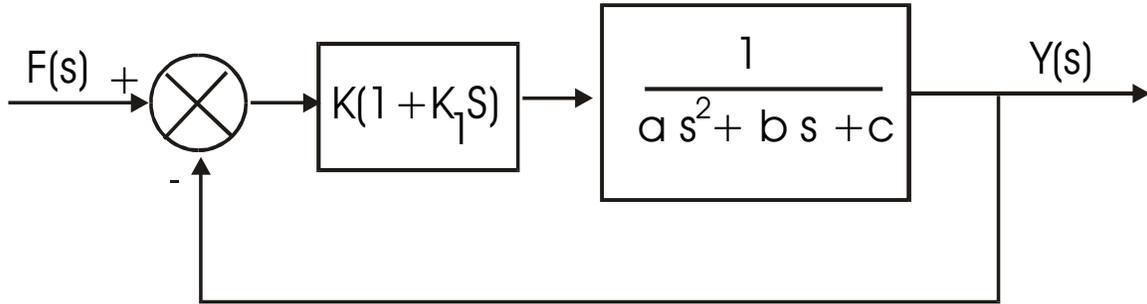
$$\text{poles} = \frac{-b \pm \sqrt{b^2 - 4a(c + K)}}{2a}$$

$\text{Error}_{ss} = 1 - \lim_{s \rightarrow 0} s T(s)$
use unit step input, $1/s$

$$= 1 - \lim_{s \rightarrow 0} s T(s)$$

$$= \frac{c}{c + K}$$

(d) Consider the following feedback scheme with a PD cascade compensator. Reduce the block diagram into a single block. Express the poles and zeros of the transfer function as a function of a, b, c, K, & K_1 . Estimate the steady state error.



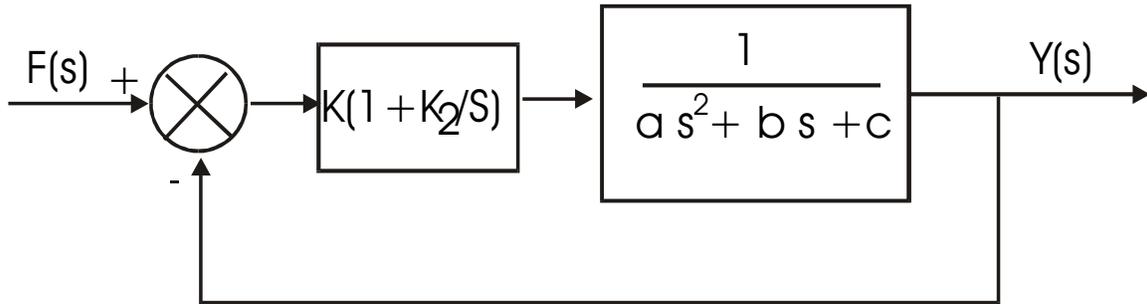
$$T(s) = \frac{K_1 K s + K}{a s^2 + (b + K_1 K) s + c + K}$$

$$\text{zeros} = -1/k_1$$

$$\text{poles} = \frac{-(b + K_1 K) \pm \sqrt{(b + K_1 K)^2 - 4a(c + K)}}{2a}$$

$$\text{Error}_{ss} = \text{same as part c, } \frac{c}{c + K}$$

(e) Consider the following feedback scheme with a PI cascade compensator. Reduce the block diagram into a single block. Express the zeros of the transfer function as a function of a, b, c, K1 and K2. What is the number of poles? Estimate the steady state error. You do not need to evaluate the pole positions explicitly. In the lab, you will be provided with values for these coefficients and the poles positions can be solved using MatLab.



$$T(s) = \frac{Ks + KK_2}{as^3 + bs^2 + (c + K)s + KK_2}$$

zeros: $-K_2$

poles: 3 roots of denominator of T(s)

$$E_{ss} = 1 - \frac{KK_2}{KK_2} = 0$$