Reviews

(1) 5 slides I showed at the begining of this course
what students can do now

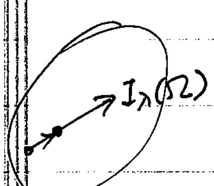
(2) presentation of solar energy & global warming

10 students — 80 minutes
what are potential topics — 8 minutes/person
6 minutes f presentati
2 minutes ~~slides~~ questi

① global energy balance
② effects of gases in the atmosphere
③ effects of particulates
④ absorpti by land & water

(3) last time
equation of radiative transfer.

$$\frac{dI_\lambda}{ds} = -(K_{a\lambda} + K_{s\lambda}) I_\lambda + K_{a\lambda} I_{b\lambda} + \frac{K_{s\lambda}}{4\pi} \int_{4\pi} \Phi(\Omega' \to \Omega) \cdot I_\lambda(\Omega') d\Omega'$$



↑ {scattered out, absorbed} ↑ emitted ↑ scattered in.

~~more~~ Boltzmann equation $f(t, \vec{r}, \vec{v})$

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_{\vec{r}} f + \vec{a} \cdot \nabla_{\vec{v}} f = \left(\frac{\partial f}{\partial t}\right)_{scattered}$$

└ Integral form.

Chapter 6 RADIATIVE TRANSFER EQUATION

In chapters 1-3, we studied radiation exchange between surfaces separated by nonparticipating medium, i.e., the medium in between does not absorb or emit radiation. Now lets turn our attention to the transfer of radiative energy through an absorbing, emitting and scattering medium. When a ray of radiation propagating through the medium, as shown in Fig. 6.1, the absorption decreases its intensity while the emission of the medium increases its intensity. Scattering is more a complicated. On one hand, scatters along the path will decrease the intensity by changing the direction of the incoming radiation. On the other hand, other scatterers may scatter radiation into the ray. In this chapter, we will first derive an equation governing the radiative intensity and then study some of its general properties.

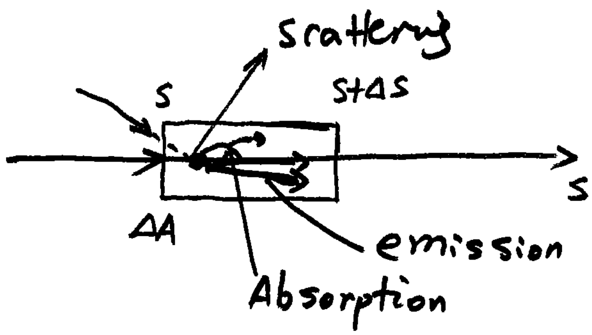


Figure 6.1 Radiation exchange in a participating medium

6.1 DERIVATION OF THE TRANSFER EQUATION

We take a step by step approach, starting from the simplest situation and gradually increasing the complexity of the problem.

6.1.1 Absorption Only

Consider a ray of radiation in a small solid angle propagating in the direction s, as shown in Fig. 6.2. We know from our discussion on EM theory that intensity decrease exponential along the path,

$$I_\lambda \sim \exp[-K_{a\lambda} s] \qquad (6.1)$$

where $K_{a\lambda}$ is the absorption coefficient at the specific wavelength. From EM theory, we know that for a homogeneous medium

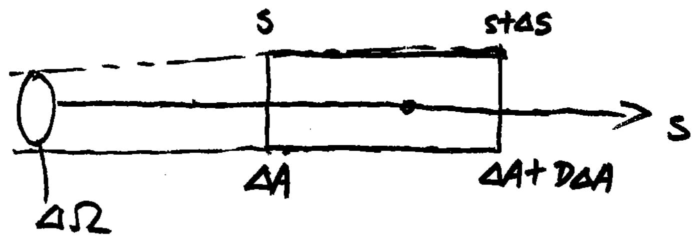


Figure 6.2 Radiation transfer through an absorbing region

$$K_{a\lambda} = \frac{4\pi\kappa}{\lambda} \tag{6.2}$$

This suggests that we could write the following differential equation for the intensity

$$\frac{dI_\lambda}{ds} = -K_{a\lambda} I_\lambda \tag{6.3}$$

In fact we could derive the above properties upon a new understanding of the physical significance of the absorption coefficient. Lets consider a balance of the radiative energy (or photon numbers). Recall that intensity is defined as the radiative energy per unit solid angle and per unit area in the direction of the ray and per unit wavelength. We could write the radiative energy flow into the region as

$(\Delta A + D\Delta A)$ $\quad\quad$ $(\Delta A + \frac{1}{2} D\Delta A)$

$$I_\lambda(s)\Delta A\Delta\Omega\Delta\lambda - I_\lambda(s+\Delta s)\Delta A\Delta\Omega\Delta\lambda = K_{a\lambda} I_\lambda(s)\Delta A\Delta\Omega\Delta\lambda\Delta s \tag{6.4}$$

Using Taylor expansion, we get the differential equation given in Eq. (6.3).† This tells us that absorption coefficient can be understood as

$$K_{a\lambda} = \lim \frac{energy\ absorbed\ in\ \Delta V}{(energy\ incident\ on\ \Delta V)\Delta s} \tag{6.5}$$

We can easily solve the Eq. (6.3) to get

† The area change $D\Delta A$ is always a higher order term. We will drop it and use $\Delta A + D\Delta A \approx \Delta A$.

$$I_\lambda(s) = I_{o\lambda} \exp\left[-\int_0^s K_{a\lambda}(s')ds'\right] \tag{6.6}$$

If the absorption coefficient is constant along the path, then

$$I_\lambda(s) = I_{o\lambda} e^{-K_{a\lambda} s} \tag{6.7}$$

From this solution, we can get the transmissivity and absorptivity of the medium as

$$\tau_\lambda' = \frac{I_\lambda(s)\Delta A \Delta\Omega \Delta\lambda}{I_{o\lambda}\Delta A \Delta\Omega \Delta\lambda} = e^{-K_{a\lambda} s} \tag{6.8}$$

and absorptivity

$$\alpha_\lambda' = 1 - e^{-K_{a\lambda} s} \tag{6.9}$$

Even though we have not consider emission, we will include it immediately. From the Kirchoff's law, we have

$$\alpha_\lambda' = 1 - e^{-K_{a\lambda} s} = \varepsilon_\lambda' \tag{6.10}$$

Now we can pose and think back. In the limit of very thin layer, Δs--0, we have

$$\alpha_\lambda' = \varepsilon_\lambda' = K_{a\lambda}\Delta s \tag{6.11}$$

This relation will help us in the following derivations.

6.1.2 Absorbing Emitting, Nonscattering Medium

Consider a slightly more complicated case, an absorbing and emitting medium without scattering, as shown in Fig. 6.3. Take a small segment and apply the radiative energy balance. We have

radiation in+radiation emitted=radiation out+radiation absorbed (6.12)

$$I_\lambda(s)\Delta A \Delta\Omega \Delta\lambda + \varepsilon_\lambda'(\Delta s) I_{b\lambda}(T)\Delta A \Delta\Omega \Delta\lambda = I_\lambda(s+\Delta s)\Delta A \Delta\Omega \Delta\lambda + \alpha_{a\lambda}(\Delta s) I_\lambda(s)\Delta A \Delta\Omega \Delta\lambda \tag{6.13}$$

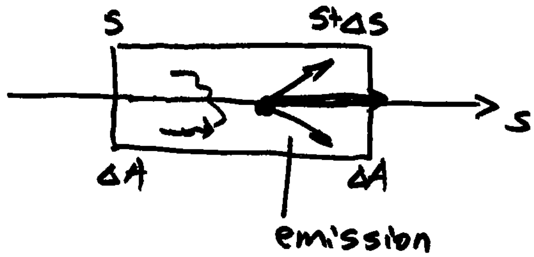


Figure 6.3 Energy balance in an ansorbing and emitting medium

Using Taylor expansion and the limit expressions for the absorptivity and emissivity, we get

$$\frac{dI_\lambda}{ds} = -K_{a\lambda} I_\lambda + K_{a\lambda} I_{b\lambda} \tag{6.13}$$

If the temperature of the medium is constant, for example, a chamber of gas which can be approximated as at constant temperature, the blackbody radiation intensity is a constant. We can then integrate the above expression and get

$$I_\lambda(s) - I_{b\lambda} = (I_{o\lambda} - I_{b\lambda})e^{-K_{a\lambda}s} \tag{6.14}$$

or

$$I_\lambda(s) = I_{o\lambda}e^{-K_{a\lambda}s} + I_{b\lambda}[1 - e^{-K_{a\lambda}s}] = \tau_\lambda' I_{o\lambda} + \varepsilon_\lambda' I_{b\lambda} \tag{6.15}$$

The first term in the last equaton represents the attenuation of intensity due to absorption and the second term represents the enhancement of intensity due to emission.

6.1.3 Scattering Only Medium

Let's temporarily neglect absorption and emission and consider the case when the medium scatter raidation, as shown in Fig. 6.4. The energy balance for a small control volume in s direction can be written as

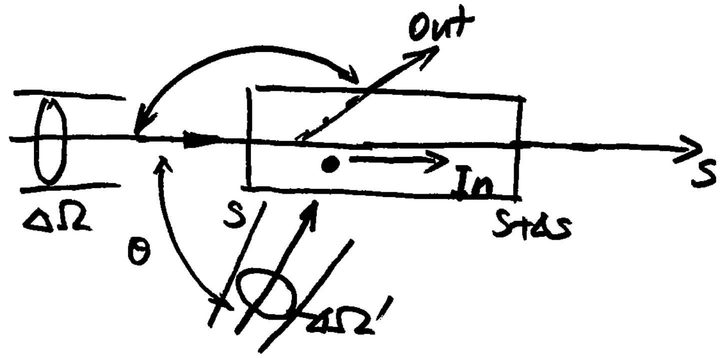


Figure 6.4 In and out scattering

Radiation into ΔV along Ω+Radiation scattered into Ω direction

=Energy leaving ΔV along Ω+Energy scattered out of Ω direction (6.16)

Let's consider the out scattering and in scattering separately.

Out Scattering. Recall we have defined scattering cross-section, efficiency, and coefficient in the previous chapter. We assume that those energy that is scattered change its direction and is no longer in the same direction as the original beam. So the intensity is lost. This lost of intensity, however should be differentiated from the lost of intensity due to absorption. Absorption converts the radiation energy into internal energy while scatter merely redistribute the radiative energy. Another comments is that the scattered energy is distributed in all over the whole solid angle. Mathematically, there is always possibility that some of the energy scattered energy is in the same direction as the incoming energy. Generally, the intensity in the incoming radiation direction is small compared to the total energy scattered in the rest of the space. One exception is the geometrical optics limit for the scattering of a particle. Recall that we showed that the extinction of radiation are due to two parts, one is due to diffraction caused by the edge of the particle and the other is due to reflection and refraction. In the geometrical optics limit, the diffraction contributes 1 to the extinction efficiency. But we said that we often neglect the scattering effects because the scatted energy is centered in the forward direction. The only scattering is due to the geometrical obstacle. For the radiative energy balance, we should use the scattering coefficient as we use the absorption coefficient, so the radiative balance equation is

$$I_\lambda(s)\Delta A\Delta\Omega\Delta\lambda = I_\lambda(s+\Delta s)\Delta A\Delta\Omega\Delta\lambda + K_{s\lambda}\Delta s I_\lambda(s)\Delta A\Delta\Omega\Delta\lambda \qquad (6.17)$$

Or

$$\frac{dI_\lambda}{ds} = -K_{s\lambda} I_\lambda \tag{6.18}$$

Recall for a cluster of particles, the scattering coefficient can be calculated from the scatter cross-section of a single particle as

$$K_{s\lambda} = \sum_i C_{si} N_i \tag{6.19}$$

In Scattering. Radiation incident from other directions may be scattered into the considered ray direction. In this case, we have to know the distribution of the scattered energy for the ray coming from direction Ω'. Recall we defined the phase function for this purpose. The phase function was defined as

$$p(\Omega' \to \Omega) = \lim \frac{\textit{energy scattered from } \Omega' \text{ into } \Omega \textit{ direction}}{\textit{energy scattered } \text{into } \Omega \textit{ if scattering were isotrpoic}} \tag{6.20}$$

From the definition, we can express then

Energy scattered from Ω'-Ω=p(Ω'-Ω)•energy scattered into Ω if scattering were isotropic

$$= p(\Omega' \to \Omega) \bullet I_\lambda' \cos\theta \Delta A \Delta\Omega' \Delta\lambda \bullet \frac{\Delta\Omega}{4\pi} K_{s\lambda} \frac{\Delta s}{\cos\theta} \tag{6.21}$$

Scattering in s due to incident in all directions= $\Delta A \Delta\Omega \Delta\lambda \bullet \frac{\Delta\Omega}{4\pi} K_{s\lambda} \Delta s \int_{4\pi} p(\Omega' \to \Omega) \bullet I_\lambda'(\Omega') d\Omega'$

Radiative energy balance gives

$$\frac{dI_\lambda}{ds} = \frac{K_{s\lambda}}{4\pi} \int_{4\pi} p(\Omega' \to \Omega) \bullet I_\lambda'(\Omega') d\Omega' \tag{6.22}$$

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6.1.4 Radiative Transfer Equation in An Absorbing, Emitting, and Scattering Medium

Combine all of the mechanisms

$$\frac{dI_\lambda}{ds} = -(K_{a\lambda} + K_{s\lambda})I_\lambda + K_{a\lambda}I_{b\lambda} + \frac{K_{s\lambda}}{4\pi}\int_{4\pi} p(\Omega' \to \Omega) \bullet I'_\lambda(\Omega')d\Omega' \qquad (6.23)$$

or

$$e_\Omega \bullet \nabla I_\lambda(\Omega) = -K_{e\lambda}I_\lambda + K_{a\lambda}I_{b\lambda} + \frac{K_{s\lambda}}{4\pi}\int_{4\pi} p(\Omega' \to \Omega) \bullet I'_\lambda(\Omega')d\Omega' \qquad (6.24)$$

6.2 RADIATIVE FLUX

Now suppose we already know the radiative intensity in all the directions at a point. For heat transfer calculations, we need to find out what are the heat flux at the same location. We recall that intensity is defined on unit solid angle while heat flux is per unit area. Lets consider the heat flux in one coordinate direction e_i. For an intensity in the e_Ω direction, the radiation energy flow across an area is

$$q_i = \int_0^\infty \int_{4\pi} \frac{I_\lambda \mathbf{e}_\Omega \bullet \mathbf{e_i} A d\lambda d\Omega}{A} = \int_0^\infty \int_{4\pi} I_\lambda \mathbf{e_W} \bullet \mathbf{e_i} d\lambda d\Omega \qquad (6.25)$$

The vector heat flux is then the component in all three coordinate directions

$$\mathbf{q} = \sum_{i=1}^{3} q_i \mathbf{e_i} \qquad (6.26)$$

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6.2.1 Divergence of Radiative Heat Flux

Once we know the heat flux, we can calculate the heat generation. This is similar to the derivation of heat conduction equations. We can take a local control volume and apply the first law, we get

$$\text{Volumetric radiative heat generation} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = \nabla \bullet \mathbf{q_r} \qquad (6.27)$$