22.02 – Introduction to Applied Nuclear Physics

Problem set # 9

Issued on Thursday May. 3, 2012. Due on Wednesday May 9, 2012

Problem 1: Solar neutrinos

Fusion reactions occur in the sun. One of the most common chain starts from the fusion of protons to achieve alpha particles. The following processes (that you need to complete with the missing components) occur:

a) $^{1}H+^{1}H \rightarrow \cdots + \cdots + \cdots$

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b) ^{1}\text{H}+\cdots \rightarrow ^{3}\text{He}+\gamma
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(note this is not a beta process, still conservation rules apply)

c) ${}^{1}\text{H}+{}^{3}\text{He} \rightarrow {}^{4}\text{He}+\cdots+\ldots$

Problem 2: Helium-8

Helium-8 is one of the most neutron-rich isotope that can be produced. Recently [Phys. Rev. Lett. 99, 252501 (2007)] it has been studied to determine its nuclear radius (shedding light on various nuclear models). In order to study the radius, He-8 was produced and then kept at rest by an atom trap (given by lasers). Helium-8 decays by a beta process: What is the reaction? What is the kinetic energy of the reaction products?

Problem 3: Beta Decays

In the decay of Cu-64 both neutrinos and antineutrinos are observed.

- a) What are the two possible beta-decay processes and products?
- **b**) Calculate the Q values for both decay modes.

Problem 4: Neutrino spectrum

The spectrum of the emitted beta particles can be computed by considering the density of states of the outgoing particles. Omitting constant parameters, the energy dependence is given by:

$$N(T_e) \propto F(Z,Q) \left[Q - T_e\right]^2 \sqrt{T_e^2 + 2T_e m_e c^2} \left(T_e + m_e c^2\right) \approx \left[Q - T_e\right]^2 \sqrt{T_e^2 + 2T_e m_e c^2} \left(T_e + m_e c^2\right)$$

where the last relationship is valid for small Z.

a) Plot the spectrum of the beta particles, assuming Q = 0.5 MeV.

(You can use your favorite mathematical software. The simplest solution is to use www.wolframalpha.com and plot the spectrum online. There's then a link at the bottom of the page to download your result as pdf.)

b) Calculate and plot the spectrum of the neutrino, assuming the same Q = 0.5 MeV.

Problem 5: Beta Decay: Selection Rule

For many beta decay processes the neutrino and electron/positron produced have zero angular momentum, l = 0. Both the neutrino and the electron (as well as their anti-particles) have intrinsic spin angular momentum $S = \frac{1}{2}$.

a) If the parent nuclide has initial spin I_i , what are the possible values of the daughter nuclide spin, I_f ?

b) If the parity of the parent nuclide is Π , what is the parity of the daughter nuclide? (note that the parity of particles such as the neutrino and beta is always given by $\Pi = (-1)^l$).

Problem 6: Neutron Elastic Scattering

We derived in class the energy loss for the neutron in an elastic scattering:

$$\Delta E/E_0 = (E_1 - E_0)/E_0 = \frac{A^2 + 1 + 2A\cos\vartheta}{(A+1)^2} - 1$$

Here ϑ is the angle between the initial neutron direction and its f nal direction in the *center of mass* frame.

a) What is the angle φ (see Fig. 49 of lecture notes) between the initial neutron direction and its f nal direction in the *lab* frame?

You should only derive the following relationship:

$$\cos\varphi = \frac{1 + A\cos\vartheta}{\sqrt{A^2 + 1 + 2A\cos\vartheta}}$$

b) When can backscattering (in the lab frame) occurr? For which isotope it never occurrs?

c) In the *center of mass* frame the scattering is isotropic, meaning that the probability of $\cos \vartheta$ is constant: $P(\cos \vartheta) = \frac{1}{2}$. Then the probability of a certain angle φ is obtained from $P(\cos \varphi)d(\cos \varphi) = P(\cos \vartheta)d(\cos \vartheta)$. What is the average $\langle \cos \varphi \rangle$?

[You can use again e.g. wolframalpha to calculate the integral, but in that case you'll need to try out some values for A to f nd a solution (e.g. try A=1,2,3 and deduce the result for any A).]

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