Problem Set 10

1. Moment of Inertia: Disc and Washer

(a) A thin uniform disc of mass M and radius R is mounted on an axis passing through the center of the disk, perpendicular to the plane of the disc. Determine the moment of inertia about the following two parallel axes that are perpendicular to the plane of the disk.



(i) An axis passing through the center of mass of the disc.

(ii) An axis passing through a point on the rim of the disc a distance ${\cal R}$ from the center.

Express your answers in some or all of the following variables: M and R.

(b) A 1 inch US Standard Washer has inner radius $r_i = 13.5$ mm and an outer radius $r_o = 31.0$ mm. The washer is approximately d = 4.0 mm thick. The density of the washer is $\rho = 7.8 \times 10^3$ kg m⁻³. Calculate the moment of inertia of the washer about an axis that is perpendicular to the plane of the washer and passes through its center of mass.



2. Compound Pulley



Consider a fixed wheel that consists of a uniform cylindrical disk with a radius R and a mass M_1 , and a second uniform cylindrical disk of radius R/2 and mass M_2 that is attached concentrically to the first disk, as shown in the top view (above right). The wheel is free to rotate about a frictionless pivot at its center. A rope is wound around the smaller disk, then around a small frictionless post. The other end of the rope is attached to a block of mass M_b so that both disks rotate together as the block falls. The downward acceleration of gravity is g. The block is released from rest.

How long does it take the block to fall a distance d? Express your answer in terms of R, M_1, M_2, M_b, g , and d as needed.

3. Suspended Rod

A uniform rod of Length L = 2.0 m and mass m = 4.0 kg is hinged to a wall at one end and suspended from the wall by a cable that is attached to the other end of the rod at an angle of $\beta = 30^{\circ}$ to the rod (see figure below). Assume the cable has zero mass. There is a contact force at the pivot on the rod. Use α to represent the angle of the pivot force (the force acting at the wall). The magnitude and direction of this force is unknown. In this problem, you will solve for the magnitude of the tension in the cable and the direction and magnitude of the pivot force.



- (a) What is the magnitude of the tension in the cable?
- (b) What angle does the pivot force make with the beam in degrees?
- (c) What is the magnitude of the pivot force?

4. Person Standing on a Hill

A person is standing on a hill that is sloped at an angle of α with respect to the horizontal. The person's legs are separated by a distance d, with one foot uphill and one downhill. The center of mass of the person is at a distance h above the ground, perpendicular to the hillside, midway between the person's feet. Assume that the coefficient of static friction between the person's feet and the hill is sufficiently large that the person will not slip.



- (a) What is the magnitude of the normal force on each foot $(N_1 > N_2)$? Express you answer in terms of some or all of the following: m, α, h, d and g.
- (b) How far must the feet be apart so that the normal force on the upper foot is just zero? This is the moment when the person starts to rotate and fall over. Express you answer in terms of some or all of the following: m, α , h and g.

5. A Cylinder Rolling in a V-Groove

A cylinder of mass m and radius R is rotating in a V-shaped groove with a constant angular velocity ω_0 . The coefficient of friction between the cylinder and the surface is μ .



What external torque must be applied to the cylinder to keep it rotating at a constant angular velocity? Express your answer using some or all of the following variables: m for the mass of the cylinder, g for the gravitational acceleration, R for the radius of the cylinder and μ for the coefficient of friction.



6. A Massive Pulley and a Block on an Incline

Consider a pulley of mass m_p and radius R that has a moment of inertia $\frac{1}{2}m_pR^2$. The pulley is free to rotate about a frictionless pivot at its center. A massless string is wound around the pulley and the other end of the rope is attached to a block of mass m that is initially held at rest on frictionless inclined plane that is inclined at an angle β with respect to the horizontal. The downward acceleration of gravity is g. The block is released from rest.

How long does it take the block to move a distance d down the inclined plane? Write your answer using some or all of the following: R, m, g, d, m_p , β .

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