1. Wave polarization: An Electromagnetic wave is the superposition of two linearly polarized waves along the $\hat{y}$ and $\hat{z}$ directions and is described by the following equation:

$$\vec{E} = \hat{y}E_0 \sin(\omega t - kx) + \hat{z}E_0 \cos(\omega t - kx). \qquad (1)$$

   (a) What is the direction of propagation of this wave?
   (b) Describe the wave's polarization.
   (c) Find the magnitude of the electric field at all points of space for all times.
   (d) An observer stands at the origin of the coordinate system.
   (e) What is the magnetic field B?

2. Spherical Wave:

$$\vec{E}(r, \theta, \phi, t) = A\frac{sin\theta}{r}[cos(kr - \omega t) - (1/kr)sin(kr - \omega t)]\hat{\phi} \qquad (2)$$

   where, $\omega/k = c$.
   (a) Show that $\vec{E}$ obeys all four of Maxwell's equations, in vacuum, and find the associated magnetic field.
   (b) Calculate the Poynting vector. Average $\vec{S}$ over a full cycle to get the intensity vector $\vec{I}$.
   (c) Integrate $\vec{I} \cdot d\vec{a}$ over a spherical surface to determine the total power radiated.

3. Poynting vector and capacitor: A current $I = dQ/dt$ delivers charge to a capacitor. This capacitor has radius a, and the plates are separated by a distance s.
   (a)Find the Poynting vector due to the electric field and the magnetic field between the capacitor plates. Give both the magnitude and the direction.
   (b) Calculate the total power, $P = \int \vec{S} \cdot d\vec{a}$, flowing into the capacitor. Given the Poynting vector found in (a), what is the correct surface to use for the integral?
   (c)Integrate this power over time. Assuming that the capacitor has charge 0 at t = 0 and has some charge level Q at a later time t, show that the total energy that flows into the capacitor is given by $U = Q^2/2C$.

4. Poynting vector and coaxial cable:

   A coaxial cable transmits DC power from a battery to a load. The cable consists of two concentric, long, hollow cylinders of zero resistance. The inner cylinder has radius a, the outer has b, and the length of both is l. The battery provides an EMF $\mathcal{E}$ between the two conductors at one end of the cable, and the load is a resistance R connected between the two conductors at the other end.

   (a) How much power is dissipated in the resistor?
   (b) What are $\vec{E}$ and $\vec{B}$ in the cable?

(c) What is the Poynting vector $\vec{S}$ in the cable?

(d) Show that $\int \vec{S} \cdot d\vec{a} = P$[from part (a)].

(e) Suppose the battery is now reversed. Does the direction of $\vec{S}$ change?

5. Radiation of from the Sun. At top of the atmosphere the average radiant flux from the Sun is $N = 1.35 \times 10^3 W/m^2$. Although this radiation consists of a spectrum of frequencies, many of the interesting properties do not depend on frequency and can therefore be calculated by using the methods described for monochromatic waves. (a) What is the average energy density in the solar radiation at the top of the atmosphere? (b) What is the average momentum density? (c) What average force would the radiation exert on a completely absorbing surface with an area of $1m^2$ oriented perpendicular to the Earth-Sun line? (d) What is the average value of E_0 in the wave?

6. Transmission line.

Two long conductors of width w and separation s form a waveguide as shown below.
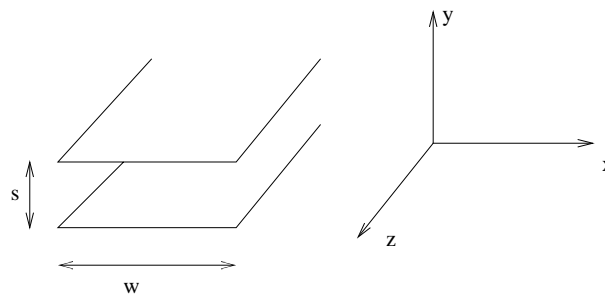

Figure 1: The waveguide.

(a) Find the inductance per unit length L_0 and the capacitance per unit length C_0 in terms of w,s and constants.

(b) One end of the transmission line is driven by a voltage $V(t) = V_0 f(zk - \omega t)$, Find the electric and magnetic fields $\vec{E}$ and $\vec{B}$ and the Poynting vector $\vec{S}$ in terms of w, s and V_0 and constants. (c) Find the impedance Z of the waveguide in terms of w, s and/or V_0 and constants.

(d) At z=10cm from the driven end of the waveguide, the waveguide changes to width $w + \Delta w$ ($\Delta w \ll w$), and separation $s + \Delta s \ll s$). What is the impedance Z' of the waveguide for $z > 10cm$? Use a Taylor expansion to express your answer as a linear function as Δs and Δw.

(e) How must Δs and Δw be related in order for $Z' = Z$?

7. Review Problem: Conductivity

A cylindrical glass rod is heated with a torch until it conducts enough current to cause a lightbulb to glow. It has a length L, a radius r, and its ends are connected to copper wires. When red hot, its conductivity varies with position x measured from the center

of the rod as $\sigma(x) = \sigma_0 L^4/x^4$.

(a) What is the resistance of this rod?

(b) When a voltage v is applied between the two ends, what is the current density $\vec{J}(x)$, and what is the steady-state electric field $\vec{E}(x)$?

(c) In steady state, what is the volume charge density $\rho(x)$ found within the rod?