Massachusetts Institute of Technology
Department of Physics
8.022 Fall 2004
Assignment 6: Equivalence of fields; Magnetic Potential; Biot-Savart law
Due date: Friday, Oct 29th

1. Purcell 6.4 Magnetic field of a hairpin shaped wire.

2. Purcell 6.13 Helmholtz coil.

3. Purcell 6.22 Magnetic Moment.

4. Purcell 6.33 Field of moving charges.

5. Magnetic field.

   (a) Find the magnetic field at the center of a square loop, which carries a steady current I. Let R be the distance from center to side.

   (b) Find the field at the center of a regular n-sided polygon, carrying a steady current I. Again, let R be the distance from the center to any side.

   (c) Check that your formula reduces to the field at the center of a circular loop, in the limit of large n.

6. Vector potential of an infinite solenoid.

   Find the vector potential of an infinite solenoid with n turns per unit length, radius R, and current I. (Hint: it is a bit hard to use Eq 35. of Purcell, since the solenoid is infinitely long here. Try thinking of

   $$\oint \vec{A} \cdot d\vec{l} = \int (\bigtriangledown \times \vec{A}) \cdot d\vec{a} = \int \vec{B} \cdot d\vec{a} = \phi \tag{1}$$

   where ϕ is the magnetic flux of $\vec{B}$ through the loop in question.)

7. Transformation of fields: A very large sheet of charge lies in the $x - y$ plane of the frame F. The charge per unit area of this sheet is σ. In the frame F', this sheet moves to the right with speed v.

   (a) What is the electric field in the rest frame (above and below the sheet)?

   (b) What is the electric field in the frame F' (above and below the sheet)?

   (c) What is the magnetic field in the frame F' (above and below the sheet)?

   (d) Show that the results of (b) and (c) are consistent with the general Lorentz transformations for electric and magnetic fields, Eq. (60) of Purcell Chapter 6.

8. Electric and magnetic forces.

   Two infinite lines of charges with charge per unit length λ_0 in their rest frame are separated by a distance d. These charges are moving in a direction parallel to their length with speed v.

(a) In the rest frame, what is the electric force per unit length that the top line feels due to the bottom line? Give both the direction and the magnitude.

(b) Repeat (a) in the lab frame.

(c) In the lab frame, what is the magnetic force per unit length that the top line feels due to the bottom line? Give both the direction and the magnitude.

(d) What is the total force per unit length in the lab frame?