Massachusetts Institute of Technology
Department of Physics
Physics 8.022 – Fall 2003
Final Examination
Monday, December 15, 2003, 09:00AM-11:00AM

Total points in the exam are 100. **ALL** problems receive **equal** points (25 each). Work on all problems. Work on problems you are most comfortable with **first.** This is a closed book and closed notes exam. An equations table will be given to you.

Problem 1

You come across a spherically symmetric electric field with the following form:

$$\begin{aligned}
\vec{E}(r) &= E_0 \left(\frac{r}{R}\right)^2 \hat{r} \qquad 0 \le r \le R \\
&= 0 \qquad R < r \le 2R \\
&= E_0 \left(\frac{r}{R} - 2\right) \hat{r} \qquad 2R < r \le 3R \\
&= E_0 \left(\frac{3R}{r}\right)^2 \hat{r} \qquad 3R < r \le 4R \\
&= 0 \qquad r > 4R
\end{aligned}$$

$\hat{r}$ is the radial unit vector in spherical coordinates.

(a) For all r, what is the charge $Q(r)$ contained within a radius r?

(b) Calculate the charge density $\rho(r)$ everywhere.

(c) Are there any surface charges in this charge distribution? If so, identify their location and give the magnitude of the surface charge density σ at each such location.

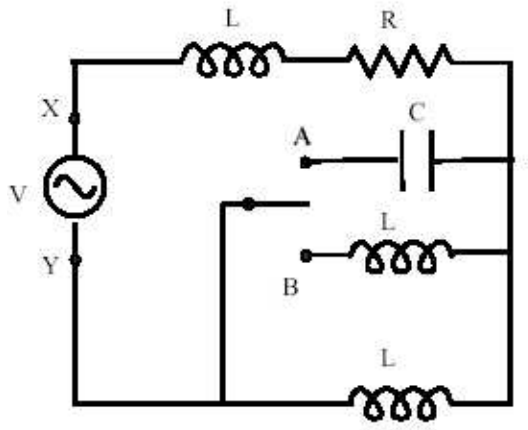
(d) The charge distribution is modified in some way. The new electric field is

$$\begin{aligned}
\vec{E}(r) &= E_0 \left(\frac{r}{R}\right)^2 \hat{r} \qquad 0 \le r \le R \\
&= 0 \qquad R < r \le 2R \\
&= E_0 \left(\frac{r}{R} - 2\right) \hat{r} \qquad 2R < r \le 3R \\
&= E_0 \left(\frac{3R}{r}\right)^2 \hat{r} \qquad 3R < r \le 7R/2 \\
&= 0 \qquad r > 7R/2
\end{aligned}$$

Compute the difference in energy between this and the old configuration, $U_{\text{new}} - U_{\text{old}}$. Was work done on the system or did the system do work?

Problem 2

The circuit illustrated in the following figure is driven by an EMF $V = V_o e^{i\omega t}$.



At time t_1, the switch is closed on A.

(a) Calculate the complex impedance Z_{XY} of the circuit.

(b) What is the non-zero frequency ω_o that maximizes the average power dissipated in the resistor R?

3) For $\omega = \omega_o$, what is the average power dissipated in the resistor R?

4) Calculate I_R, the current flowing through the resistor R when $\omega = \omega_1 = 1/\sqrt{2CL}$. What is the phase between I_R and V when $\omega = \omega_1 = 1/\sqrt{2CL}$? Is I_R leading or lagging V?

At time t_2, the switch is closed on B.

5) What is the complex impedance Z_{XY} of the circuit now?

6) How does $|V_R|$ depend on the frequency ω? Draw a graph.

Problem 3

The electric field of a traveling electromagnetic wave in vacuum is given by
$\vec{E} = 10(3\hat{x} + 4\hat{z})sin(ky + 2\pi 10^9 t)$ statvolt/cm.
All given and requested numerical quantities are in CGS units.

(a) What is the direction of propagation of this wave?

(b) What is the frequency f of this wave?

(c) What is the wavelength λ of this wave?

(d) What is the amplitude of the electric field E_o?

(e) What is the amplitude of the corresponding magnetic field B_o?

(f) Find the corresponding magnetic field $\vec{B}$ to the given $\vec{E}$ above.

(g) What is the polarization of this wave?

(h) Find the Poynting vector $\vec{S}$ for this electromagnetic wave.

(i) A totally absorbing photodetector of flat surface A lies perpendicularly to the direction of propagation. What is the pressure the wave exerts as impinges on it? What is the intensity of the electromagnetic radiation collected by the photodetector?