8.04: Quantum Mechanics
Massachusetts Institute of Technology

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Final Exam Formula Sheet

Fourier Transform Conventions:

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \; e^{ikx} \tilde{f}(k) \qquad \tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \; e^{-ikx} f(x)$$

Delta Functions:

$$\int_{-\infty}^{\infty} dx\, f(x)\, \delta(x-a) = f(a) \qquad \delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk\, e^{ikx}$$

$$\delta(x) = \left\{ \begin{array}{ll} 0 & x \neq 0 \\ \infty & x = 0 \end{array} \right\} \qquad \delta_{mn} = \left\{ \begin{array}{ll} 0 & m \neq n \\ 1 & m = n \end{array} \right\}$$

Operators and the Schrödinger Equation:

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2} \qquad [\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x} \qquad i\hbar \frac{\partial}{\partial t} \psi(x,t) = \hat{E}\, \psi(x,t)$$

$$\hat{E} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \qquad E\, \phi_E(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \phi_E(x) \; + \; V(x)\, \phi_E(x)$$

Common Integrals:

$$\int_{-\infty}^{\infty} dx\, e^{-x^2} = \sqrt{\pi} \qquad (f|g) = \int_{-\infty}^{\infty} dx\, f(x)^*\, g(x)$$

For an infinite square well with $0 \leq x \leq L$:

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin(k_n x) \qquad (\phi_n|\phi_m) = \delta_{mn}$$

$$k_n = \frac{(n+1)\pi}{L} \qquad E_n = \frac{\hbar^2 k_n^2}{2m}$$

Continuity Condition for $V(x) = W_o \delta(x-a)$:

$$\phi_E(a^+) = \phi_E(a^-) \qquad \phi'_E(a^+) - \phi'_E(a^-) = \frac{2mW_o}{\hbar^2} \phi_E(a)$$

Physical Constants:

$$\hbar \simeq 6.6 \cdot 10^{-16}\ eV \cdot s \qquad m_e = 5 \cdot 10^5\ eV/c^2 \qquad c = 3 \cdot 10^8\ m/s$$

The Probability Current:

$$\mathcal{J}(x,t) = \frac{i\hbar}{2m}\left(\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial x}\right)$$

Definition of the S-matrix and the scattering phase:

$$\begin{pmatrix} B \\ C \end{pmatrix} = S \begin{pmatrix} A \\ D \end{pmatrix}, \qquad t = |t|\, e^{-i\varphi}, \qquad T = |t|^2$$

Raising and Lowering Operators for the 1d Harmonic Oscillator ($\beta^2 = \hbar/m\omega$):

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\frac{1}{\beta}\hat{x} + i\frac{\beta}{\hbar}\hat{p}\right)\ , \qquad \hat{a}^\dagger = \frac{1}{\sqrt{2}}\left(\frac{1}{\beta}\hat{x} - i\frac{\beta}{\hbar}\hat{p}\right)\ , \qquad \left[\hat{a}, \hat{a}^\dagger\right] = 1$$

Normalization and Orthonormality of 1d HO wavefunctions :

$$\phi_n(x) = A_n e^{-x^2/2\beta^2} H_n\!\left(\frac{x}{\beta}\right) \qquad A_n = (2^n n!\, \beta\sqrt{\pi})^{-1/2} \qquad (\phi_n|\phi_m) = \delta_{nm}$$

Laplacian in Spherical Coordinates.

$$\vec{p} = -i\hbar\vec{\nabla} \qquad \vec{\nabla}^2 = \frac{1}{r}\partial_r^2 r + \frac{1}{r^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)$$

Angular Momentum Operators in Spherical Coordinates:

$$\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}\ , \qquad \hat{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

Angular Momentum Commutators

$$[\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x\ , \qquad [\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y\ , \qquad [\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z\ , \qquad [\hat{L}_i, \hat{L}^2] = 0$$

Angular Momentum Raising and Lowering Operators

$$\begin{aligned} \hat{L}_+ &= \hat{L}_x + i\hat{L}_y = \hbar e^{+i\phi}\left(i\cot\theta\,\partial_\phi + \partial_\theta\right) \qquad [\hat{L}_z, \hat{L}_\pm] = \pm\hbar\,\hat{L}_\pm\ . \\ \hat{L}_- &= \hat{L}_x - i\hat{L}_y = \hbar e^{-i\phi}\left(i\cot\theta\,\partial_\phi - \partial_\theta\right) \qquad [\hat{L}^2, \hat{L}_\pm] = 0\ . \end{aligned}$$

First Few Spherical Harmonics

$$Y_{0,0} = \sqrt{\frac{1}{4\pi}}\ , \qquad Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta\ , \qquad Y_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}\ , \qquad (Y_{lm}|Y_{l'm'}) = \delta_{ll'}\delta_{mm'}\ .$$