## 15.066j System Optimization and Analysis Summer 2003 Professor Stephen C. Graves

## Group Problem Set 5

## This problem set is due at the end of recitation on 17<sup>th</sup> July.

**Problem 1**: An oil company produces three brands of oils: Regular, Multigrade and Supreme. Each brand of oil is composed of one or more of four crude stocks, each having a different viscosity index. The relevant data is:

| Crude Stock | Viscosity index | Cost (\$/barrel) | Supply per day (barrels) |
|-------------|-----------------|------------------|--------------------------|
| 1           | 20              | 7.1              | 1000                     |
| 2           | 40              | 8.5              | 1100                     |
| 3           | 30              | 7.7              | 1200                     |
| 4           | 55              | 9.00             | 1100                     |

Each brand of oil must satisfy a minimum standard for viscosity index and each brand thus sells at a different price. The relevant data concerning the three brands are:

| Brand      | Minimum Viscosity Index | Selling Price (\$/barrel) |
|------------|-------------------------|---------------------------|
| Regular    | 25                      | 8.50                      |
| Multigrade | 35                      | 9.00                      |
| Supreme    | 50                      | 10.00                     |

One of three scenarios can occur:

|            | Low demand (prob. $= 0.2$ ) | Med. demand (prob. $= 0.5$ ) | High demand (prob. $= 0.3$ ) |
|------------|-----------------------------|------------------------------|------------------------------|
| Regular    | 1800                        | 2000                         | 2100                         |
| Multigrade | 1300                        | 1500                         | 1600                         |
| Supreme    | 650                         | 750                          | 800                          |

The company cannot sell more than its demand. Also any unused crude oil is disposed off at no cost. The decision of buying crude stock is taken well in advance and thus the company does not know what scenario holds at the time of buying crude stock. However at the time of production of oil the company knows the exact demand.

How much of crude stock must the company buy in order to maximize its expected profit?

**Problem 2**: The Auto company of America (ACA) produces 4 types of cars: subcompact, compact, intermediate and luxury. ACA also produces trucks and vans. Vendor capacities limit total production capacity to at most 1,200,000 vehicles per year. Subcompacts and compacts are built together in a facility with a total annual capacity of 620,000 cars. Intermediate and luxury are built together in a facility with a total annual capacity of 400,000 cars; and the truck/ van facility has a capacity of 275,000. Profit margins and fuel efficiencies are summarized below:

| Туре         | Profit Margin (\$/vehicle) | Fuel Efficiency (MPG) |
|--------------|----------------------------|-----------------------|
| Subcompact   | 150                        | 40                    |
| Compact      | 225                        | 34                    |
| Intermediate | 250                        | 15                    |
| Luxury       | 500                        | 12                    |
| Truck        | 400                        | 20                    |
| Van          | 200                        | 25                    |

The company expects the demand level to be one of the three: **high demand** (with probability = 0.2), **medium demand** (with probability = 0.5) and **low demand** (with probability = 0.3). Note that the company does not know which scenario holds until the selling season arrives, long after it has taken its production decisions.

The demands for these scenarios is:

| Туре         | High Demand (in '000) | Medium Demand(in '000) | Low Demand (in '000) |
|--------------|-----------------------|------------------------|----------------------|
| Subcompact   | 700                   | 600                    | 550                  |
| Compact      | 500                   | 400                    | 350                  |
| Intermediate | 400                   | 300                    | 250                  |
| Luxury       | 325                   | 225                    | 175                  |
| Truck        | 425                   | 325                    | 275                  |
| Van          | 200                   | 100                    | 50                   |

If the company ends up making more cars than the demand, it can sell its excess stock through a discount channel and earn half the normal profit; for example surplus compact cars would yield profits of \$112.5/ vehicle.

The average fleet fuel efficiency must be atleast 27 MPG.

What should the production plan of the company be so that its:

- a) Expected revenue is maximized
- b) Minimum revenue is maximized