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*Electromechanical Dynamics*

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## Appendix C

# SUMMARY OF PARTS I AND II AND USEFUL THEOREMS

### IDENTITIES

$$\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \mathbf{A} \cdot \mathbf{B} \times \mathbf{C},$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi,$$

$$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B},$$

$$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B},$$

$$\nabla(\phi\psi) = \phi \nabla\psi + \psi \nabla\phi,$$

$$\nabla \cdot (\psi \mathbf{A}) = \mathbf{A} \cdot \nabla\psi + \psi \nabla \cdot \mathbf{A},$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B},$$

$$\nabla \cdot \nabla\phi = \nabla^2\phi,$$

$$\nabla \cdot \nabla \times \mathbf{A} = 0,$$

$$\nabla \times \nabla\phi = 0,$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A},$$

$$(\nabla \times \mathbf{A}) \times \mathbf{A} = (\mathbf{A} \cdot \nabla)\mathbf{A} - \frac{1}{2}\nabla(\mathbf{A} \cdot \mathbf{A}),$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A})$$

$$\nabla \times (\phi \mathbf{A}) = \nabla\phi \times \mathbf{A} + \phi \nabla \times \mathbf{A},$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}.$$

## THEOREMS

$$\int_a^b \nabla \phi \cdot d\mathbf{l} = \phi_b - \phi_a.$$

A diagram showing a closed loop path starting at point 'a' and ending at point 'b'. The path is indicated by arrows pointing clockwise. A small differential vector element  $d\mathbf{l}$  is shown along the path, with its tip at a point on the curve between 'a' and 'b'.

Divergence theorem

$$\oint_S \mathbf{A} \cdot \mathbf{n} da = \int_V \nabla \cdot \mathbf{A} dV$$

A diagram showing a volume  $V$  enclosed by a surface  $S$ . A small differential area element  $\mathbf{n} da$  is shown on the surface  $S$ , with its normal vector  $\mathbf{n}$  pointing outward.

Stokes's theorem

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot \mathbf{n} da$$

A diagram showing a closed loop  $C$  enclosing a surface  $S$ . A small differential area element  $\mathbf{n} da$  is shown on the surface  $S$ , with its normal vector  $\mathbf{n}$  pointing outward.